The Doctrine and the Origins of Dialectic Causal Theory of Action Economics

M. M. Khoshyaran

Abstract. The objective of this paper is to discuss the doctrine and the origins of Dialectic-Causal Theory of Action,(DCTA) economics. The idea is to relate syntax and its' dialectic element, syntax transformations, to action. Action is a physical manifestation of movement that causes displacement in order to reach a destination. Action is assumed to be the causality of the occurrence of demand, and supply, and all the side effects, surplus, equilibrium, and growth. There are (14) axioms in this doctrine. These axioms are fundamental as the building blocks of the (DCTA) economics. The axioms are constructed as tools used to define and formulate the fundamental concepts of the (DCTA) economics, namely, demand, and supply.

M.S.C. 2010: 42A20, 42A32.
Key words: DCTA economics, Causal Theory of Action, axioms, syntax, syntax transformation, Dialectic, causality, action, causal space, causal tensors, hyper planes, hyper triangles, demand, supply, surplus, equilibrium, growth.

1 Introduction

The objective of this paper is to introduce the doctrine and the origins of Dialectical-Causal Theory of Action, (DCTA) economics,[1], [2] . The doctrine is based on (14) axioms. All these axioms are based on the concept of the (DCTA). This means that all physical activities that end in the realization of a demand | supply are originated from a mental process that is outwardly expressed by a form of a syntax. The dialectic, [3], [4], [5], [6], [7], [9], [8], [10], [11] aspect is the result of various permutations and combinations of the different elements of the syntax that is the underlying causality of the ensuing physical movements towards realizing demand| supply for a product. Thus the origin of the (DCTA) is syntax and its' evolution, and the causal action induced by syntax and thus the combinations and permutations of action or (physical movements)due to the dialectic aspect of syntax. Action is assumed to be the causality of the occurrence of demand| supply and all the side effects, such as consumer|producer surplus, utility, equilibrium, and demand| supply growth. The (14) axioms are used as operators acting on a syntax, and or on a modified syntax (syntax modified by adding a dialectic attachment). Variational syntax could be considered as a physical object

,

representing a thought process that causes action. As operators act on a syntax, they cause action to evolve into more sophisticated forms. Action produced by syntax is used to construct demand| supply forms.

Syntax, [12], [13], [14], [15], [16], [17], [18], [19], can be defined and the relationships among various syntax can be identified using simpler or more basic object which are these (14) axioms which normally are presented as unproved propositions. In the context of the (DCTA), these axioms are not arbitrary, but rather are fundamental concepts that can be proved in order to show self- consistency meaning that they possess a certain logic and as such can be demonstrated in a systematic fashion. The axioms are non-redundant due to their logical basis. These axioms are independent categories that are complete and sufficient for the development of the (DCTA) economics, and are only valid and used to construct unique sets of entities such as the (DCTA) demand and supply which are the fundamentals of an economic system. This creates an incidence to redefine other related elements such as (decision making, consumer| producer surplus, utility, equilibrium, and consumer| producer growth), [38]. Decision making is a thought process and is considered equivalent to a syntax. In contrast to the classical economic theory, utility, and consumer surplus define a demand curve and consequently, the supply curve as a response to this demand. The equilibrium ensues. Consequently, positive change in the equilibrium indicates growth; the (DCTA) economics reformulates all the above factors as a function of a thought manifestation represented by syntax, represented by various geometries as hyperplane manifolds that are mapped onto a 3 dimensional space defining causal action, [20], [21], [22], [23],[24], [25], [26], [27], [28], [29], [30], [31], [32], [33], [34], [35] represented by spheroids. Syntax, and action, come together with the two basic elements of economic theory, which are quantity demanded, and produced, generally denoted by (q), and price per quantity consumed, and produced denoted as (p). The n-dimensional causal space (S, A), where (S) stands for syntax, and (A) stands for action translated to a (12) sided cuboid, [36], [37] that is used to connect action to (q), and (p).

The axioms of DCTA economics is constructed as a back ground condition to be used in defining , and formulating the (DCTA) system of economics. (DCTA) economics is a system of economics that considers dialectic reasoning and CTA (Causal Theory of Action) as the building blocks in constructing the most fundamental economic concepts,(demand, supply, decision making, consumer| producer surplus, utility, equilibrium, and consumer| producer growth). The (DCTA) axioms are listed here.

Axiom 1. Causality: Each individual possesses a thought process which represents a decision making process. A thought process or the decision making process is represented by a collection of syntax. Syntax is the physical representation of a thought process that is the cause of an individual taking an action in order to fulfill a decision taken by the same individual. Thus causality in this context refers to the syntax producing actions that reflect the intentions of an individual. Syntax is made up of segments that are added together in a coherent manner in order to convey a certain logic. Different segments of a syntax are called causal sets, causal groups, and causal categories. It is the combinations and or permutations of causal sets that are members of different causal groups that constitute different causal categories that give a syntax a specific meaning. Each combination and or permutation of the different segments of syntax indicate various causalities of actions.

Axiom 2. Syntax: Syntax is the cause of action or physical movement. A syntax is a structure that allows for the expression of a certain logic or rationale. In the context of the DCTA economics, this logic relates to an economic activity such as consumption or production. Therefore, the subsets are chosen from specific groups, which belong to specific categories. Namely groups and categories that indicate displacement in space, using physical movements.

Axiom 3. Evolution of action: Any syntax can evolve from its' original form through transformation. Transformation refers to enhancement of the original syntax by adding a combination or permutation of new or different subsets from the subsets already used, that fall into groups belonging to categories that do not necessarily contain action inducing segments, but give more details or depth to the original syntax. The addition of syntax transformers leads to causality for more complicated physical movements or action. This is an evolution in action, since it does not create abrupt change in the physical movements, but rather, induces complimentary movements that help in performing the economic acts of consumption or production.

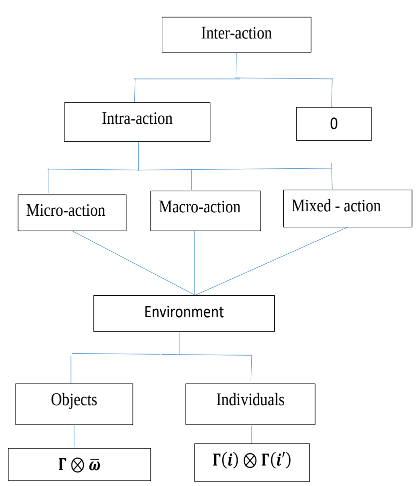
Axiom 4. Dialectic: dialectic refers to dialectic reasoning, is embedded in the syntax transformation. Syntax transformation is the attachment of segments of subsets of groups of different categories to the original syntax. Transformation adds more details to the original syntax, it also adds more depth and creates perception that either complements the perception derived from the original syntax in a positive direction or creates an argument that can affect the perception of the original syntax in a negative direction (creating doubts) that can cause modifications of the original syntax, and modifications in the actions required to achieve an economic activity, (consumption, production).

Axiom 5. Dialectic equilibrium: equilibrium in the causal context, is defined as the region where the causal hyper-plane demand and the causal hyper-plane supply share in common. This region is in a hyper-space and is of trapezoidal shape. The dialectic equilibrium occurs if there exists transformations in the original syntax that cause modifications in the perceived actions. In other words, required actions are modified, which causes change of the angle of the hyper-plane demand and the angle of the hyper-plane supply causing tilt or rotation in of the hyper-trapezoidal shape of the equilibrium. The main issue is that new physical movements leading to action do not cause discontinuities. Discontinuity refers to either an abrupt change in the shape or the existence of holes. The hyper-trapezoidal shapes retain their aspect in the case of no discontinuities, and change aspect in the case of the existence of discontinuities; translates into twists or holes on the surface. Thus geometrical shapes created by the new movements retain a regularity in shape, (hyper-planes, and hyper-surfaces without discontinuities).

Axiom 6. Forms of action: This axiom states that there are different types of actions. Different types of actions are: 1) micro actions that refer to physical movements done within a small space, (small region). 2) Macro actions refer to physical movements within a large space (large region). 3) Mixed actions refer to any combination of micro-macro actions, mainly, the union, $\cup$; or intersection $\cap$ of different types of actions. Proportional mixed actions, and probabilistic mixed actions are subsets of the group of mixed actions in the category of transitional actions.

Axiom 7. Transitional actions: The term transitional actions refers to the different types of actions (described in axiom 6) that can be performed in zones outside of the original zone indicated. Micro action transition is micro actions that are performed within a large space (large region). This means that micro-actions are repeated a number of times within a large space in order to cover the large zone. Similarly, macro actions can be performed within a small space, by the adjustment of the macro actions such as taking a fractional macro actions, performed in a small zone. Micro-macro action transition can be described as including/replacing micro or macro transitional actions mixed with regularly defined micro-macro actions. This can be extended to subsets of mixed actions: proportional or probabilistic mixed actions. Special cases occur based on the assignment of one proportion equal to zero, or one probability equal to zero that creates singular micro/micro transitional, or singular macro/transitional macro actions. Transitional actions are caused by syntax transformations, (due to the transformation operators), which transforms the different elements of a syntax. Any syntax transformation is mapped onto the action space, as transitional actions. Geometrically, a transition in action is represented as the change in the shape of an action manifold. This means tilted, twisted, or elongated manifolds, without discontinuities.

Axiom 8. Interactions causing intra-actions. The process of inter-action leading to intra-action is demonstrated in the flow chart below:



Flow chart of interaction-leading to intra-action

In the flow chart inter-action refers to the interaction among micro actions or macro actions, or mixed actions. Two possibilities exist: 1) interactions do not lead to interacting with other elements such as objects or individuals. In this case interaction ends at this stage. 2) Interactions that necessitate dealing with objects or individuals existing in the environment. Interaction causing interactions happen either in a small zone or in a large zones that include objects or other individuals. In the case of intra-action with objects in the environment, it is represented as an action tensor denoted by ($\mathbf{\Gamma}$), is multiplied by ($\overline{\omega}$), ($\mathbf{\Gamma} \otimes \overline{\omega}$). ($\overline{\omega}$) refers to the angle of interaction while manipulating environmental objects. The angle $\overline{\omega}$) changes depending on the topology of action. In the case of intra-action with other individuals, it is represented as the multiplication of the action tensors of the an individual, (i), with another individual, (i'), represented as ($\mathbf{\Gamma}(i) \otimes \mathbf{\Gamma}(i')$). It is assumed that other individuals either reciprocate actions, or modify their responsive actions.

Thus interaction is a reference to the topology of action, (the manner of interaction). Interaction is a combined algebraic form of the interaction among action types, that defines the geometrical forms of action-interaction. Intra-action is defined as the extended consequence of interaction among different action types. For example, a micro-macro interaction of an individual that results in interacting with other individuals, or objects and is extended into interacting with groups of individuals or objects within (immediate and extended) the existing physical environment shared by all. The topological aspect of the interaction and intra-action are expressed in tensor functional terms.

Axiom 9. Persistency of action, (PA): In axiom 7, the transitional actions are continuous actions with respect to physical movements, or work. Physical movements refer to various directions of body movement involved in performing work translated to action. There is no break in the transition from one action type to another, or within different variations of the same type of action. Topologically this states that differential functions representing the transitional process with respect to work done within zones of small (r), or large (R) radii, (γ_r, γ_R), where (γ_r) denotes work done within radius (r), and $(\gamma_r = \gamma_r(I_r, v_r, F_r))$. (I) denotes the intensity of action, (v) denotes the speed of action, and (F) denotes the frequency of action. (γ_R) denotes work done within within radius (R), and $(\gamma_R = \gamma_R(I_R, v_R, F_R))$. (γ_r), and (γ_R) are non-zero tensor functions of physical elements that are found in a syntax, denoted by (ξ) and its' transformations, denoted by (Δ), (ξ, Δ) such as the intensity of action (I), (v) the speed of action, and (F) the frequency of action. (PA) is represented as an tensor operator with elements the differentials of the physical elements category for $((\xi, \Delta) \rightarrow \Gamma(\gamma_r, \gamma_R))$, where $(\Gamma(\gamma_r, \gamma_R)$ represents the mapping of $((\xi, \Delta))$, syntax, and its' transformations tensor onto action tensor, $(\Gamma(\gamma_r, \gamma_R))$. Geometrically, the (PA) is represented as tensor differential manifolds, $(\partial \mathbf{\Gamma}(\gamma_r, \gamma_R))$. A tensor differential manifold is represented as $(\partial \mathbf{\Gamma} = \frac{\partial(\Gamma(\gamma_r, \gamma_R)}{\partial(I) \partial(v) \partial(F)})$. Thus (PA) is represented as a differential manifold $(\partial \mathbf{\Gamma})$, constituted by differential tensors, $(\frac{\partial(\Gamma(\gamma_r, \gamma_R)}{\partial(I) \partial(v) \partial(F)})$ considered as fibre bundles indicating the direction of action tensor,$(\Gamma(\gamma_r, \gamma_R))$.

Axiom 10. Force of action,(FA): The (FA) refers to change in the position of a syntax tensor manifolds, $(\mathbb{H}(\xi, \Delta))$ that causes change in the position of the action tensor manifold, $(\mathbf{\Gamma}(\gamma_r, \gamma_R))$. Let the change in the position be represented by the change in the position of $(\mathbb{H}(\xi, \Delta))$ from a horizontal position to a different angle. Since $(\mathbb{H}(\xi, \Delta) \rightarrow \mathbf{\Gamma}(\gamma_r, \gamma_R))$, then let $(\mathbb{H}_{\overline{\varpi}}(\xi, \Delta) \rightarrow \mathbf{\Gamma}_{\overline{\varpi}}(\gamma_r, \gamma_R))$, where $(\overline{\varpi})$ represents the angle of distortion from the horizontal position. $(\overline{\varpi})$ becomes a tensor of angle change. $(\mathbb{H}_{\overline{\varpi}}(\xi, \Delta))$ is the change in the position of $(\mathbb{H}(\xi, \Delta))$ by angle $(\overline{\varpi})$. This is mapped onto the corresponding action tensor $(\mathbf{\Gamma}(\gamma_r, \gamma_R))$ represented as $(\mathbf{\Gamma}_{\overline{\varpi}}(\gamma_r, \gamma_R))$. Change in the position angle $(\overline{\varpi})$ is caused due to the gravitational force caused by the following actions: 1) dialectic equilibrium; 2)transitional action; 3) inter-action causing intra-action; 4) persistency of action, axioms 5,7,8,9.

Topologically, the (FA) causing torsion can be represented by a tensor representing change in the position angle $(\overline{\varpi})$, represented by a tensor function of the algebraic representation of the (4) axioms, (5,7,8,9),that act on $(\mathbb{H}(\xi, \Delta) \rightarrow \mathbf{\Gamma}(\gamma_r, \gamma_R))$, to $(\mathbb{H}_{\overline{\varpi}}(\xi, \Delta) \rightarrow \mathbf{\Gamma}_{\overline{\varpi}}(\gamma_r, \gamma_R))$.

Theorem 1.1. *The mapping from $(\mathbb{H}(\xi, \Delta))$ to $(\mathbf{\Gamma}(\gamma_r, \gamma_R))$; $(\mathbb{H}(\xi, \Delta) \rightarrow \mathbf{\Gamma}(\gamma_r, \gamma_R)$ is an isomorphic mapping. Therefore, any change in a syntax manifold is an isomorphic mapping onto the action manifold; $(\mathbb{H}_{\overline{\varpi}}(\xi, \Delta) \rightarrow \mathbf{\Gamma}_{\overline{\varpi}}(\gamma_r, \gamma_R))$, where $(\overline{\varpi})$ is an angle of change. .*

Proof. The proof consists of two parts: 1) There exists a mapping from $(\mathbb{H}(\xi, \Delta))$ to $(\mathbf{\Gamma}(\gamma_r, \gamma_R))$. 2) This mapping is isomorphic.
1) Each syntax (ξ) contains either explicitly, or implicitly a dormant action as long as the action is not accomplished. Thus let $(\xi = \xi(\gamma_r, \gamma_R))$ be a syntax containing an embedded action(s). 2) The process of going from a dormant (implied) action in a syntax $(\xi(\gamma_r, \gamma_R))$ to a performed action $(\Gamma(\gamma_r, \gamma_R))$ is a direct mapping. This means that $(\xi(\gamma_r, \gamma_R) = \Gamma(\gamma_r, \gamma_R))$. Action, $(\Gamma(\gamma_r, \gamma_R))$ is a direct manifestation of syntax, since $(\Gamma(\gamma_r, \gamma_R)$ is a physical form of the syntax, $(\xi(\gamma_r, \gamma_R))$, $(\forall)$ $(\gamma_r \in \xi(\gamma_r, \gamma_R))$, and $(\forall)$ $(\gamma_R \in \xi(\gamma_r, \gamma_R))$, $(\exists)$ $(\Gamma(\gamma_r, \gamma_R)$, such that $(\Gamma(\gamma_r, \gamma_R) = (\gamma_r \oplus \gamma_R))$. Both (γ_r), and (γ_R) are spheroids of radii (r), and (R), where $(\gamma_r = I_r^2 dI_r + v_r^2 dv_r + F_r^2 dF_r)$, and $(\gamma_R = I_R^2 dI_R + v_R^2 dv_R + F_R^2 dF_R)$. (I) is the intensity of action, (v) is the speed of action, and (F) is the frequency of action. Let a transformed syntax be denoted as (ξ, Δ). (ξ, Δ) is mapped onto distorted action represented by tilted spheroids within the angle of rotation, $(\bar{\omega})$. Therefore, $((\xi, \Delta) = \Gamma(\gamma_r, \gamma_R) \otimes \bar{\omega} = \Gamma'(\gamma_r, \gamma_R) = (I_r^2 dI_r + v_r^2 dv_r + F_r^2 dF_r) \otimes \bar{\omega} \oplus (I_R^2 dI_R + v_R^2 dv_R + F_R^2 dF_R) \otimes \bar{\omega})$. □

Geometrically, axiom 10, is demonstrated as torsion, that causes distortion in the causal tensor syntax manifold, $(\mathbb{H}(\xi, \Delta))$, and is mapped onto action manifold,$(\mathbf{\Gamma}(\gamma_r, \gamma_R))$. Torsion is a change in the position, (twist, or tilt) of $(\mathbb{H}(\xi, \Delta))$, and consequently, change in the position of the movements representing action. Since the syntax manifold, is a planar manifold, a change in the position could be taken as the occurrence of curvatures in $(\mathbb{H}(\xi, \Delta))$.

Axiom 11. Dialectic of Power, (DP): this axiom relates to axiom 10, the force of action, (FA), represented as torsion. The (DP) is related to the evolution of torsion.

Torsion evolves by taking a varied directions and different rotations. Thus (DP) is topologically represented as a tensor operator with elements the differentials of the angle of rotation, ($\bar{\omega}$), with respect to the axes of action, (I), (v), and (F). expressed as a sheaf operator, ($\mathfrak{D}$), where ($\mathfrak{D} = \frac{\partial \bar{\omega}}{\partial I_{r,R}} \otimes \frac{\partial \bar{\omega}}{\partial I v_{r,R}} \otimes \frac{\partial \bar{\omega}}{\partial F_{r,R}}$). Thus, the transformed syntax manifold is mapped onto an action manifold multiplied by the sheaf operator, ($\mathbb{H}(\xi, \Delta) \to \mathbf{\Gamma}(\gamma_r, \gamma_R) \otimes \mathfrak{D}$). Geometrically, ($\mathbf{\Gamma}(\gamma_r, \gamma_R) \otimes \mathfrak{D}$), represents the rotation of spheroids of action around one of the axes of rotation, (I), (v), and (F).

Axiom 12. Gravitational Centrality,(GC): this axiom relates to how action gravitates towards specific centers. Axiom 12, relates to axiom 11, (DP), where ($\Gamma(\gamma_r, \gamma_R) \otimes \mathfrak{D}$), the causal action tensor, ($\Gamma(\gamma_r, \gamma_R)$), is multiplied by the (DP) operator, ($\mathfrak{D}$). The action manifold ($\mathbf{\Gamma}(\gamma_r, \gamma_R)$), impacted by the (DP) operator, ($\mathfrak{D}$), is curved, and tilted in a way that there exists regions in the causal action manifold,($\mathbf{\Gamma}$) where there exists points at which the causal action manifold is at its' maximum position; ie. ($\mathbf{\Gamma}(\gamma_r, \gamma_R) \otimes \mathfrak{D} = 0$). In the context of the (DCTA) economics, these centers of gravitation are either thought centers (represented outwardly by a transformed syntax), (ξ, Δ) mapped onto the action manifold ($\mathbf{\Gamma}$), or syntax transformations that introduces spatial centers, (physical environment) that require modifications in ($\mathbf{\Gamma}$) acted through isomorphic mapping. Geometrically, the centers of gravitation of action are regions of the causal action manifold, (causal due to ($\mathbb{H}(\xi, \Delta)$)), where the sheaf of fibre bundles given as the (DP) operator contains differential vectors at a maximum point of the causal action manifold, ($\mathbf{\Gamma}(\gamma_r, \gamma_R)$).

Axiom 13. Pressure Zones, (PZ): Axiom 13 expresses the probability of the occurrence of axioms 11, and 12. Topologically, this axiom introduces a probability tensor denoted by ($\mathfrak{P}$), that gives the probability of the occurrence of the elements of the tensor operator ($\mathfrak{D}$). Let the tensor operator (PZ) be denoted as ($\mathfrak{P}_3$). The tensor operator ($\mathfrak{P}_3$) is formulated as: ($\mathfrak{P}_3 = \mathfrak{D} \otimes \mathfrak{P}$), where ($\mathfrak{P}$), is a tensor of size (i,j) written as ($\mathfrak{P} = \sum_j e_j.\rho_i^j$); ($\mathbf{e} = (I, v, F)$), represents the axes of action, and ($\rho = pr(\bar{\omega}_e|\bar{\omega})$) is the probability of the occurrence of distortion, (existence of the angle of rotation) in the direction of one of the axes given that rotation has occurred. Therefore, ($\mathbf{\Gamma}_{\mathfrak{P}_3}(\gamma_r, \gamma_R) = \mathbf{\Gamma}_{\overline{\omega}, \mathfrak{D}}(\gamma_r, \gamma_R) \otimes \mathfrak{P}_3$). Geometrically, (PZ)s are curved regions with the tendency of the causal action manifold ($\mathbf{\Gamma}(\gamma_r, \gamma_R)$)having an angle of rotation tensor ($\overline{\omega}$), and a center of gravity tensor, ($\mathfrak{D}$). The causal aspect of the occurrence of (PZ) is a transformed syntax (ξ, Δ)such that the text includes the prevalence of a certain type of an action such as an emphasis on micro actions versus macro actions over other types of action (transitional action)mapped onto causal action manifolds with pressure zones, ($\mathbb{H}(\xi, \Delta) \to \mathbf{\Gamma}(\gamma_r, \gamma_R)$).

Axiom 14. Physical Environment, (PE): axiom 14, refers to transformation in a syntax, (ξ, Δ) such that it includes special features (special causal set of the (Noun) groups of a fixed category) in the environment of an individual,that requires transitional actions of a specific form. This category is used as a transformation syntax operator to modify any initial syntax. Modified syntax maps onto transitional actions. Axiom 14, includes axioms 11-13. Topologically, axiom 14, is expressed as the application of all three operators ($\overline{\omega}$), ($\mathfrak{D}$), and ($\mathfrak{P}_3$) to the action tensor ($\Gamma(\gamma_r, \gamma_R)$) as follows: ($\Gamma_{PE}(\gamma_r, \gamma_R) = \Gamma(\gamma_r, \gamma_R) \otimes \overline{\omega} \otimes \mathfrak{D} \otimes \mathfrak{D}$). Geometrically, (PE) is represented by the causal action manifold, $((\mathbf{\Gamma}(\gamma_r, \gamma_R))$, normally assumed as hyperplanes in higher dimensions that have special curved region(s) given by $(\Gamma_{PE}(\gamma_r, \gamma_R))$.

The (14) axioms of the (DCTA) economics, will be discussed in a rigorous mathematical format in a later section after the sections on syntax, and action, followed by the mathematical representation of the (14) axioms of the (DCTA) economics. This is followed by a section on (DCTA) economic indicators, (demand, supply, equilibrium, consumer surplus, and growth) are represented in the final section.

The precondition for the (DCTA) economics is the idea of incidence and connection. The first move is to relate syntax, (ξ) to action, (Γ). This means that any syntax representation corresponds to a type of action,$(\xi \to \Gamma(\gamma_r, \gamma_R))$, and thus the causality between (ξ) to action, $(\Gamma = \Gamma(\xi, \Delta))$. The action tensor is called causal action since the origin of the action is a mapping from a syntax and its' transformation to action, $((\xi, \Delta) \to \Gamma(\gamma_r, \gamma_R))$, and $(\Gamma(\gamma_r, \gamma_R) = \Gamma(\xi, \Delta))$. The incidence and the connection criteria in this case are the relationship between quantity demanded (q) of a product, and the price (p) of a product, and action, $(\Gamma(\xi, \Delta))$. In other words, the incidence and the connection criteria guarantees the change from (q) being the result of some non-measurable thought activity to a systematic way of measuring this thought process, which is action; that can be defined and measured with metrics that are already in use. Thus (q) can be defined as a function of action, $(\Gamma(\xi, \Delta))$, $(q \to q(\xi, \Delta) = f_q(\Gamma(\xi, \Delta))_q)$, where $((\Gamma(\xi, \Delta))_q)$ is action that results in demand for a product, and $(f_q(\Gamma(\xi, \Delta))_q)$ is a hyperplane function of action ending in quantity demanded, (q). The quantity produced is denoted as $(q_{\mathfrak{P}})$ and is given as $(q_{\mathfrak{P}} \to q_{\mathfrak{P}}(\xi, \Delta) = f_{q_{\mathfrak{P}}}(\Gamma(\xi, \Delta))_{q_{\mathfrak{P}}})$, where $((\Gamma(\xi, \Delta))_{q_{\mathfrak{P}}})$ is action that results in supply of a product. The causal syntax relating to (q), corresponds to (2) thought processes: 1) an induced thought process, indicating $((\Gamma(\xi, \Delta))_q)$, action induced by a thought process towards quantity demanded (q), and 2) the form of action,$(f_q(\Gamma(\xi, \Delta))_q)$ hyperplane. Similarly, price can be defined as a function of action, $(\Gamma(\xi, \Delta))$, There are (2) types of prices, price per unit product acceptable to a consumer, denoted as $(p \to p(\xi, \Delta) = f_p(\Gamma(\xi, \Delta))_q)$, and the price per unit of production(representing the desirable price per unit of production) denoted as $(p \to p(\xi, \Delta) = f_p(\Gamma(\xi, \Delta))_{\mathfrak{P}}))$, where $((\Gamma(\xi, \Delta))_{\mathfrak{P}}))$ is production, $(\mathfrak{P})$ related action that results in setting price for a product , and $(f_p(\Gamma(\xi, \Delta))_{\mathfrak{P}}))$ is a hyperplane function of action ending in price setting, (p). Both (q), and (p), can be found from an immeasurable thought process to the one measured with metrics relating to action. Described this way both quantity demanded (q), and price (p) are points in the Syntax, (Ξ), Action, (Γ), (Ξ, Γ) causal space. It can be shown that any point in the causal space (Ξ, Γ) has the incidence and connection properties.

Another precondition for the (DCTA) economics is the existence postulate. The incidence, connection, and existence criteria are to be demonstrated in the Causal space,(Ξ, Γ). The causal space,(Ξ, Γ) is a space made of causal cuboids,(a construction resembling a cube) of dimension of at least (12). The cuboid is called causal, because its' occurrence depends on syntax, and its' transformations. The causal cuboid consists of 12 axes. Each axis constitutes a basis used to build demand and supply hyperplanes. Thus a minimum of (12) axes make up the base of the causal space (Ξ, Γ). Thus the dimension of the causal space, (Ξ, Γ) is $(\Re^{(n+m)})$, where (n), and (m) relate to action types. The causal space, (Ξ, Γ) can be expanded by adding extra cuboids, thus the causal space can expand to $(12^{(n+m)})$, where $((n+m))$ refers to the number of cuboids in the (Ξ, Γ) space. Each axis of the cuboid is at an angle with another axis. This angle is strictly greater than (0) and less than (180) degrees. To determine any quantity demanded (q), and price per unit of a product, (p) based on the type of action, (3) axes of the causal cuboid are used. The three axes are the quantity demanded,$(q(\xi, \Delta))$, and unit price $(p(\xi, \Delta))$, and any of the remaining (10) action axes, $((\Gamma(\xi, \Delta))_i; i = 3, \cdots, 10)$ of the cuboid. Thus at any event, there exists (3) angles denoted as (ϕ, θ, ψ), where $((0 \leq \{\phi, \theta, \psi\} \leq 180); i = 1, \cdot, 24)$. Each of the (3) angles, (ϕ, θ, ψ), are between(0) and (180) degrees. Orthogonal axes are the special case of the axis position. The existence of non-orthogonal axes due to causality means that the cuboids exist in a non-Euclidean space. Therefore, the causal space (Ξ, Γ) is a non-Euclidean space made up of cuboids as kernels, or (genomes). The axis of the causal cuboid are uneven. This means that each action tensor has a different number of physical movements. Causal cuboids are the basis of the causal space (Ξ, Γ). Thus each side of the causal cuboid constitutes a basis. The bases are not orthogonal, but are at angles. A causal cuboid is demonstrated in Figure 1, below.

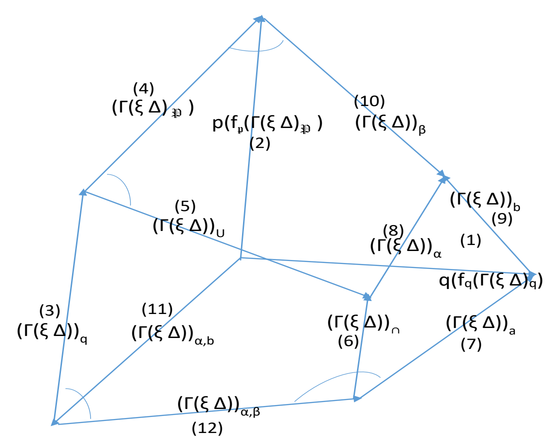


Figure 1. A Causal Cubiod

Each axis of the causal cubiod is a tensor. There are (2) axes tensors,1) one is the quantity demanded $(q = f_q(\Gamma(\xi, \Delta))_q)$, which is a hyperplane function of individual actions that end in a demand for a product. 2) the unit price tensor axis, $(p = f_p(\Gamma(\xi, \Delta))_{\mathfrak{P}}))$, is a hyperplane function of actions relating to production, $(\mathfrak{P})$ that ends in unit price setting for a product. These (2) axes are dependent on action, (physical movements or work, (γ_r, γ_R). Throughout the paper, one finds a distinction between action and work. Work refers to physical movements in either (γ_r) or (γ_R) in circular zones of radius (r), or (R) respectively. Action is a cumulative representation of work that includes work (γ_r) and work (γ_R), thus the given notation $(\Gamma(\gamma_r, \gamma_R))$.

It is assumed that $(\Gamma(\gamma_r\gamma_R) = \Gamma(\xi, \Delta))$ which means that $((\xi, \Delta) \rightarrow (\gamma_r, \gamma_R))$, and $(\Gamma(\gamma_r\gamma_R) = \Gamma(\xi, \Delta))$. In general, one obtains the following mapping between a syntax, and its' transformation and action, $((\xi, \Delta) \rightarrow \Gamma(\gamma_r\gamma_R))$.

The syntax space, (Ξ, Γ), is a non-Euclidean space allowing for both linear (affine), and non linear forms of syntax, and its' transformation, (ξ, Δ). The metric used in this space is the degree of a syntax,(d). The metric (d) is defined as the difference between the predicates with a transformation, $(P_{\mathsf{d}}(\xi, \Delta))$, and the basic syntax $(P_{\mathsf{d}}(\xi))$, formulated as $(\mathsf{d} =\mid P_{\mathsf{d}}(\xi, \Delta) - P_{\mathsf{d}}(\xi) \mid)$. Syntax and transformation, (ξ, Δ) is isomorphic to action, $(\Gamma(\xi, \Delta))$ which is a non-Euclidean space of spheroids represented as the axes of the cuboids in the multiplicative space (Ξ, Γ). Action is described in a syntax, given the degree of the syntax, (d). Thus the metric in the spheroid action space is given by the same type of formulation, denoted as (C), and formulated as $(\mathsf{C} =\mid \Gamma'(\gamma_r\gamma_R \mid - \mid \Gamma(\gamma_r\gamma_R \mid)$, where $(P_{\mathsf{d}}(\xi, \Delta) \rightarrow \Gamma'(\gamma_r\gamma_R)$, is an isomorphic mapping of the predicates of a syntax with transformations, to an action type $(\Gamma'(\gamma_r\gamma_R)$,and $(P_{\mathsf{d}}(\xi) \rightarrow \Gamma(\gamma_r\gamma_R)$ is the isomorphic mapping of the basic syntax to action type $(\Gamma(\gamma_r\gamma_R)$.

Aside from the two axes (q), and (p), all other axes of the cuboid are the combinations and permutations of action, $(\Gamma(\gamma_r\gamma_R) = \Gamma(\xi, \Delta))$. All axes of action, $(\Gamma = \Gamma(\xi, \Delta))$ are shown in Figure 2. The permutations and combinations of action, (Γ) are as follows. In Figure 2, action, (Γ) is split into two types of action, one relating to consumption, or quantity demanded denoted as (Γ_q) and one relating to production, $(\Gamma_{\mathfrak{P}})$, which is related to pricing strategies. In Figure 2, the first two axes of the causal cuboid are omitted since the emphasis is on the combinations, and permutations of actions that lead to consumption $(q = f_q(\Gamma(\xi, \Delta))_q)$, and price, $(p = f_p(\Gamma(\xi, \Delta))_{\mathfrak{P}}))$. $(\Gamma(\gamma_r\gamma_R) = \Gamma(\xi, \Delta))$ is due to isomorphic mapping from syntax to action. $((\Gamma(\xi, \Delta))_q)$ is action resulting in quantity demanded, and $((\Gamma(\xi, \Delta))_{\mathfrak{P}})$ is action relating to production activity that determines the unit price of a product. The branching in Figure 2, starts with axis 3 of the cuboid. The first branch in Figure 2, relates to axis 3,$((\Gamma(\xi, \Delta))_q)$, action ending in (q). Branch 2, is the axis 4 of the cuboid, is action relating to production, $((\Gamma(\xi, \Delta))_{\mathfrak{P}})$ that determines the unit price of a product. The next branch is axis 5, is a combination of action denoted as $((\Gamma(\xi, \Delta))_{\cup} = (\Gamma(\xi, \Delta))_q \oplus (\Gamma(\xi, \Delta))_{\mathfrak{P}})$, where $((\Gamma(\xi, \Delta))_{\cup})$ relates to the combination of two types of actions, one relating to demand, and one relating to production.

The following branch is axis 6, is a permutation of actions denoted as $((\Gamma(\xi, \Delta))_{\cap} = (\Gamma(\xi, \Delta))_q \otimes (\Gamma(\xi, \Delta))_{\mathfrak{P}})$, where $((\Gamma(\xi, \Delta))_{\cap})$ relates to the permutation (refers to tensor product) of the two types of actions. The following branch is axis 7, is denoted as $((\Gamma(\xi, \Delta))_a == a \odot (\Gamma(\xi, \Delta))_q)$, where $((\Gamma(\xi, \Delta))_a)$ is action that takes into account a sub-total of action relating to demand, and (a) is a weight (importance) assigned to the action type,$((\Gamma(\xi, \Delta))_q)$. The next branch, is axis 8, is a permutation of axis 7, and is denoted as $((\Gamma(\xi, \Delta))_\alpha = \alpha\dot{(}\Gamma(\xi, \Delta))_q)$, where $((\Gamma(\xi, \Delta))_\alpha)$ is a probabilistic action that considers the probability that $((\Gamma(\xi, \Delta))_q)$ occurs and is determinant in producing demand for a product, and (α) is the probability coefficient relating to action types, $((\Gamma(\xi, \Delta))_q)$. $((\Gamma(\xi, \Delta))_\alpha)$ represents a sub-total of actions relating to demand, since the most probable segment of action can determine demand, (q). Branch indicating axis 9, is denoted as $((\Gamma(\xi,\Delta))_b = b \odot (\Gamma(\xi,\Delta))_{\mathfrak{P}})$,where $((\Gamma(\xi,\Delta))_b)$ is the sub-total of action relating to production that is considered as the determinant of the unit price of a product, and (b) is the weight or importance assigned to the action type, $((\Gamma(\xi,\Delta))_{\mathfrak{P}})$. Branch indicating axis 10, is denoted as $((\Gamma(\xi,\Delta))_\beta = \beta \odot (\Gamma(\xi,\Delta))_{\mathfrak{P}})$,where $((\Gamma(\xi,\Delta))_\beta)$ is a probabilistic action that considers the probability that $((\Gamma(\xi,\Delta))_{\mathfrak{P}})$ occurs and is determinant in production that sets the unit price for a product, and (β) is the probability coefficient relating to action types, $((\Gamma(\xi,\Delta))_{\mathfrak{P}})$. $((\Gamma(\xi,\Delta))_\beta)$ represents a sub-total of actions relating to production, since the most probable segment of action can determine production, $(\mathfrak{P})$ which leads to setting a unit price of a product. Branch indicating axis 11, is denoted as $((\Gamma(\xi,\Delta))_{a,b} = a \odot (\Gamma(\xi,\Delta))_q \oplus b \odot (\Gamma(\xi,\Delta))_{\mathfrak{P}})$, where $((\Gamma(\xi,\Delta))_{a,b})$ is a combination of axes (7), and (9). Branch indicating axis 12, is denoted as $((\Gamma(\xi,\Delta))_{\alpha,\beta} = \alpha \odot (\Gamma(\xi,\Delta))_q \otimes \beta \odot (\Gamma(\xi,\Delta))_{\mathfrak{P}})$, where $((\Gamma(\xi,\Delta))_{\alpha,\beta})$ is a permutation of axes (8), and (10).

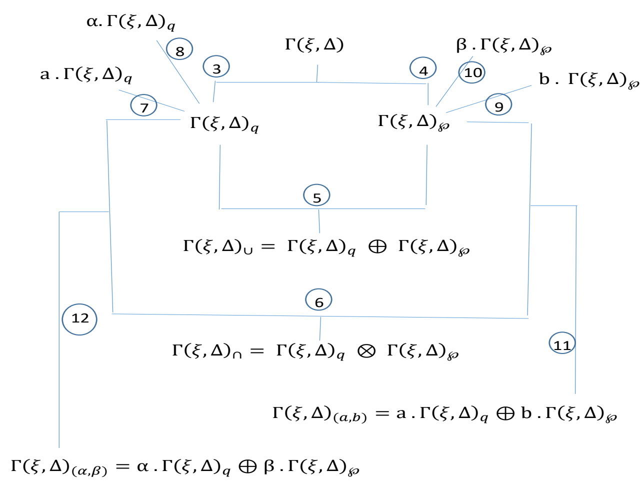


Figure 2. Combinations and Permutations of Action

The cubiod represents the basis of the causal space (Ξ,Γ), where (Ξ) denotes syntax, and (Γ) denotes action. Given the causal cubiod the incidence and the connection criteria as well as the existence postulate can be demonstrated. Theorem 2, and Theorem 3 give formal statements of incidence, and connection criteria, and existence postulate.

Theorem 1.2. *Any pair ($(q = f_q(\Gamma(\xi,\Delta))_q), (p = f_p(\Gamma(\xi,\Delta))_{\mathfrak{P}}))$),lies on a tensoral hyperplanes of the causal space (Ξ,Γ).*

Theorem 1.3. *If ($(q = f_q(\Gamma(\xi,\Delta))_q))_1, (p = f_p(\Gamma(\xi,\Delta))_{\mathfrak{P}})_1$), and ($(q = f_q(\Gamma(\xi,\Delta))_q))_2$, $(p = f_p(\Gamma(\xi,\Delta))_{\mathfrak{P}})_2$), are 2 distinct points on a line on a tensoral hyperplanes of the causal space (Ξ,Γ), then there exists at least one point ($(q = f_q(\Gamma(\xi,\Delta))_q))_3, (p = f_p(\Gamma(\xi,\Delta))_{\mathfrak{P}})_3$)) that is not on line ($(q = f_q(\Gamma(\xi,\Delta))_q))_1, (p = f_p(\Gamma(\xi,\Delta))_{\mathfrak{P}})_1$), ($(q = f_q(\Gamma(\xi,\Delta))_q)_2), (p = f_p(\Gamma(\xi,\Delta))_{\mathfrak{P}})_2$).*

Proof. The proof that applies to the 2 Theorems, 1.2, 1.3 can be constructed by demonstrating the existence of all the required elements, such as points, lines, and planes. Given the causal space (Ξ, Γ), and a causal cubiod as is shown in Figure 3, a reasonable proof can be constructed. As is shown in Figure 3, three axes are chosen for the purpose of demonstration. These axes are the quantity consumed (q), axis, the price, (p) axis, and actions resulting in demand or consumption for a product, axis (3). Let $(((\Gamma(\xi, \Delta))_q)_1, (p(\Gamma(\xi, \Delta))_{\mathfrak{P}})_1)$ be a point on axis (3), where $((\Gamma(\xi, \Delta))_q)_1)$ represents action that leads to demand, and $((p(\Gamma(\xi, \Delta))_{\mathfrak{P}})_1)$ represents action that indicates an individual's price acceptability. The point on axis 3, is projected on axis (1), the demand axis, by vector $(\vec{A})$. This is due to the fact that each of the chosen axis are manifolds and there exists an isomorphic projection from one manifold to the another, since the vector $(\vec{A})$, intersects the demand axis (q), at point ($(q* = f_q(\Gamma(\xi, \Delta))_q))_1)$, given $(p = f_p(\Gamma(\xi, \Delta))_p)_1)$, demand is a causal consequence of action, given by (f_q). From the same point on axis (3), an isomorphic projection, is represented by vector $(\vec{B})$, that intersects the price axis (p) at point ($(p* = f_p(\Gamma(\xi, \Delta))_q))_1)$, given$((p = f_p(\Gamma(\xi, \Delta))_p)_1)$. The one-to-one projection from point (q*), to point (p*) is demonstrated by vector $(\vec{C})$. The isomorphic mapping is based on Theorem 1.1. Each point on the causal action cuboid belongs to an element of a syntax, and its' transformations, $((\xi, \Delta))$, $((\xi, \Delta) \to \Gamma(\gamma_r, \gamma_R))$, where $(\Gamma(\gamma_r, \gamma_R) = \Gamma(\xi, \Delta))$ implies isomorphic mapping and $(\Gamma(\gamma_r, \gamma_R))$ can be written as $(\Gamma(\xi, \Delta))$. The three vectors, $(\vec{A}, \vec{B}, \vec{C})$ constitute a triangular hyperplane of demand. Similarly, from axis (3), a triangular supply hyperplane is obtained. Triangular hyperplanes of supply and demand can be obtained for mixed action axes. The existence of points, lines, and planes are demonstrated.

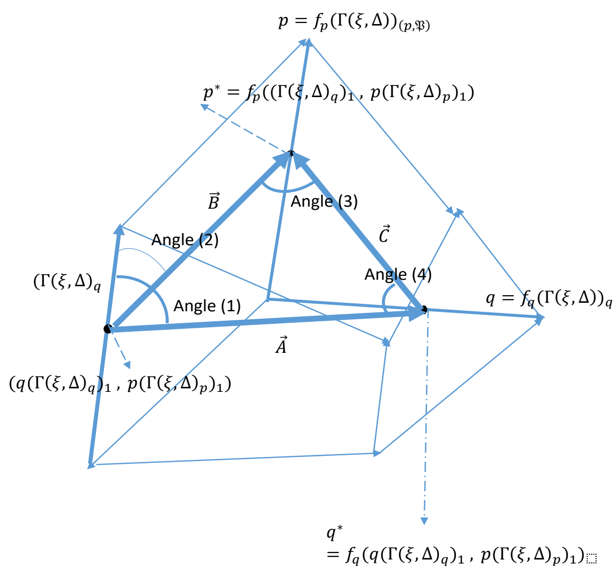


Figure 3. Demonstration of the existence of planes

$\square$

In Figure 3, the angles (1), determines the point of contact between vector, $(\vec{A})$, and the third axis, and angle (2) determines the point of contact between vector $(\vec{B})$, and the third axis respectively. Both angles (1), and (2) are between $(0 \leq (\psi, \theta, \phi) \leq 2\pi)$. The function (f_q) can be formulated as $(f_q = a.e^{-min(((\Gamma(\xi,\Delta))_q)_1,(p(\Gamma(\xi,\Delta))_p)_1)})$, where (a) is a coefficient of the exponential function with minimum exponent. The function $(f_p = b.e^{-max(((\Gamma(\xi,\Delta))_q)_1,(p(\Gamma(\xi,\Delta))_p)_1)})$, where (b) is the coefficient of the exponential function with maximum exponents.

The rotation of vector $(\vec{C})$ around the (q), and the (p) axes is given by matrix (B) as:

$$B = \begin{array}{c} \\ q \\ p \\ \Gamma_3 \end{array} \begin{array}{c} \begin{array}{ccc} \quad q \quad & \quad p \quad & \Gamma_3 \end{array} \\ \begin{pmatrix} cos(\frac{\psi^1}{2\pi}) & sin(\frac{\psi^1}{2\pi}) & 0 \\ -sin(\frac{\psi^2}{2\pi}) & cos(\frac{\psi^2}{2\pi}) & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array}$$

where angle (3) in Figure 3, is denoted as (ψ^1), and angle (4) is denoted as (ψ^2). (B) is an (3×3) matrix. The rotation of vector $(\vec{A})$ around the axes (q), and (Γ_3) is given by matrix (C)as:

$$C = \begin{array}{c} \\ q \\ p \\ \Gamma_3 \end{array} \begin{array}{c} \begin{array}{ccc} \quad q \quad & p & \quad \Gamma_3 \end{array} \\ \begin{pmatrix} cos(\frac{\theta}{2\pi}) & 0 & sin(\frac{\theta}{2\pi}) \\ 0 & 1 & 0 \\ -sin(\frac{\theta}{2\pi}) & 0 & cos(\frac{\theta}{2\pi}) \end{pmatrix} \end{array}$$

where angle (1) in Figure 3, is denoted as (θ). (C) is an (3×3) matrix. The rotation of of vector $(\vec{B})$ around the axes (p), and (Γ_3) is given by matrix (D)as:

$$D = \begin{array}{c} \\ q \\ p \\ \Gamma_3 \end{array} \begin{array}{c} \begin{array}{ccc} q & \quad p \quad & \quad \Gamma_3 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & cos(\frac{\phi}{2\pi}) & sin(\frac{\phi}{2\pi}) \\ 0 & -sin(\frac{\phi}{2\pi}) & cos(\frac{\phi}{2\pi}) \end{pmatrix} \end{array}$$

where angle (2) in Figure 3, is denoted as (ϕ). (D) is an (3×3) matrix. Figure 3, is chosen as an example of vector rotation along an axis, (Γ_3). Similar matrices exist between any two axes of the causal cuboid, where action axes are denoted as, $(\Gamma_i; i = 3, 4, \cdots, 12)$. Thus in general the matrices (B,C,D), become $((B_i, C_i, D_i, \forall i = 1, \cdots, 12))$.

Each of the 12 causal action axes of the causal cuboid is capable of rotation, and transformation. Causal axis rotation occurs within a defined reference frame. Transformation is an act that translates a causal action from one frame of reference to another frame of reference. Each of the 12 axes of the causal cuboid is a tensor that is defined within a frame of reference. The frame of reference for all axes of the cuboid are the intensity of action, (I), the speed of action, (v), and the frequency of action, (F), (I,v,F). There are two frames of reference, one relates to actions within a zone of radius (r), designated as ($(I, v, F)_r$) designated as (1) for the sake of simplicity. The second frame of reference relates to actions within a zone of radius (R), designated as ($(I, v, F)_R$), designated as (2). Each axis of the causal cuboid can rotate within the same frame of reference and move to new reference frame through linear transformations. This process is shown in a diagram in Figure 3+.

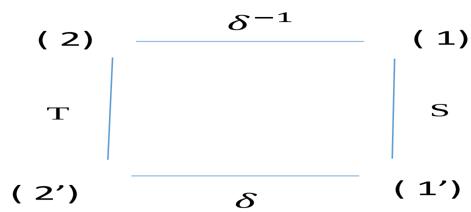


Figure 3+. Transformation diagram

In Figure 3+, a transition from (2) to (1) occurs due to a linear transformation from (2) to (1), using (δ^{-1}), where ($\delta^{-1} = (A_i)^{-1} \otimes \Gamma_i(2)) = (1)$), and ($A_i$) is a coefficient in ($\Re^3$) of a diagonal matrix of size (3×3), and ($A_i)^{-1}$) is a diagonal matrix with diagonal entries, ($\frac{1}{A_i}$). (A_i) corresponds to relations that relate to an area of large radius (R), an example of such activities are (walking, jogging, taking public transport), etc. ($\Gamma_i(1) = \Gamma_i(I_r, v_r, F_r)$) represents the coordinate of the causal axes in the reference frame (1). The transformation from the reference frame (1) to (1') is done by a linear transformation from (1) to (1'), using (S), where ($S = a_i \otimes \Gamma_i(1) = (1')$). Matrix ($a_i$) corresponds to actions that relate to an area of small radius (r), examples of such activities are (working on computers, making an object, walking around the habitat, etc). The transformation from the reference frame (1') to (2') is done by a linear transformation from (1') to (2'), using (δ), where ($\delta = (a_i \otimes \Gamma_i(1)) \otimes B_i = (2')$), where ($B_i \neq A_i$) is a coefficient in ($\Re^3$) of a diagonal matrix of size (3×3), and corresponds to the relations that relate to an area of large radius (R). (2') represents a rotation of the reference frame, (2). The final transformation is from the reference frame (2) to the reference frame (2') using a linear transformation (T). The linear transformation, (T) is formulated as ($T = (a_i \otimes B_i \otimes A_i)^{-1} \otimes \Gamma_i(1) \otimes \Gamma_i(2)$). This transformation takes into account both the rotation from the reference frame (1) to (1'), then the rotation from the reference frame (2) onto (2'). Two transformations are performed, one from the reference frame (2) to (1), and the reference frame (2) to (1') to (2'). Both causal actions ($\Gamma_i(1)$), and ($\Gamma_i(2)$) in the two reference frames are diagonal matrices of size (3×3), with diagonal entries given for one reference frame as an example:

$$\Gamma_i(1) = \begin{array}{c} \\ I \\ v \\ F \end{array} \begin{array}{c} \begin{array}{ccc} \quad I \quad & \quad v \quad & \quad F \quad \end{array} \\ \begin{pmatrix} \Gamma_i(I_r) & 0 & 0 \\ 0 & \Gamma_i(v_r) & 0 \\ 0 & 0 & \Gamma_i(F_r) \end{pmatrix} \end{array}$$

Both causal actions ($\Gamma_i(1)$, and ($\Gamma_i(2)$)) exist in a 3D Euclidean space. Causal actions are a homeomorphic mappings from an $(n+m)$-dimensional conceptual (non-Euclidean) space (ξ, Δ) to a 3D Euclidean space, $((I_r, v_r, F_r) \times (I_R, v_R, F_R))$.

2 Syntax and causal sets

The origins of the DCTA economics lies in syntax. Syntax is considered as a physical representation of a mental process that results in action. In this case action refers to physical movements that relate to an economic such as purchasing and production activities. Syntax is the product of a linguistic analysis of phrases used in common every day communication. A syntax system, consists of a conceptual class of formal objects, and a conceptual class of predicates, can be refered to as the basic predicates before any transformations. Each predicate is associated with a natural number called degree. A basic syntax consists of an ordered number of predicates (already with a given degree number), [39]. Let syntax be denoted by (ξ), and let $(\xi = \xi(X_1, \cdots, X_n)))$, where $(X_1, \cdots, X_n)$ are the names of the formal objects in different groups, each group having different categories. belonging to either type 1, or type 2 in Figure 4, and thus, let (ξ) be a notation for an n-argument verbs, assigning a basic predicates of degree (n). The parenthesis indicates that (ξ) is applied to the formal objects $(X_1, \cdots, X_n)$. If the syntax system, has (m_n) predicates, where $(n = 1, 2, \cdots)$, then for any number of predicate $(m_n > 0)$, and for some predicate number $(k = 1, 2, \cdots, m_n)$, let (ξ_n^k), be the (k)th predicate of degree (n), and let $(\xi_n^k(X_1, \cdots, X_n))$, be the (k)th predicate applied to $(X_1, \cdots, X_n)$ arguments (objects).

Given $(\xi_n^k(X_1, \cdots, X_n))$, the syntax transformation , (Δ) is an operator acting on (ξ_n^k), such that $(\Delta(\xi_n^k) = \xi_N^K)$, where the Kth predicate is $(K \neq k)$, and $(N > n)$. The transformation operator adds more arguments or objects to the original phrase,$(\Delta(\xi_n^k(X_1, \cdots, X_n)) = \xi_N^K(X_1, \cdots, X_n) \wedge (Y_1, \cdots, Y_J))$, where $(\wedge)$ is concatenation, given as $(X_i\{Y_j\})$, or $(P(X_i\{Y_j\}))$, or $(C(X_i\{Y_j\}))$, $(i = 1, \cdots, m_n)$, and $(j = 1, \cdots, M)$, then the degree (N) is $(N = 1, 2, \cdots, m_{n+M})$. $(P(X_i\{Y_j\}))$ is the permutation of the concatenation of (Y), with (X), and $(C(X_i\{Y_j\}))$ is the combination of the concatenation of (Y), with (X). The degree of a syntax specifies (3) variables corresponding to action $(\Gamma(\gamma_r, \gamma_R))$. The action is given in (ξ, Δ). The degree of (ξ, Δ) relates to the intensity of action, (I), the speed of action, (v), and the frequency of action, (F). Both (γ_r), and (γ_R) are also given in the content of the syntax.

The transformation causes dialectic in logic. Dialectic is embedded in syntax transformation (Δ). Dialectic in general is the introduction of doubt in a phrase such that it opens alternative possibilities. Thus the certainty over the basic version changes to a probability due to the enhancement of the basic syntax. The implied actions that were considered as a logical interpretation of a phrase, (ξ) become probable interpretations in (ξ, Δ), and thus each action is a probable action, $(\bar{\alpha} \times \Gamma(\gamma_r, \gamma_R))$ where $(\bar{\alpha} = (\alpha, \beta), (\alpha', \beta'))$ is the probability of the occurrence of action type $(\Gamma(\gamma_r, \gamma_R))$. Figure (4-) represents the dialectic process.

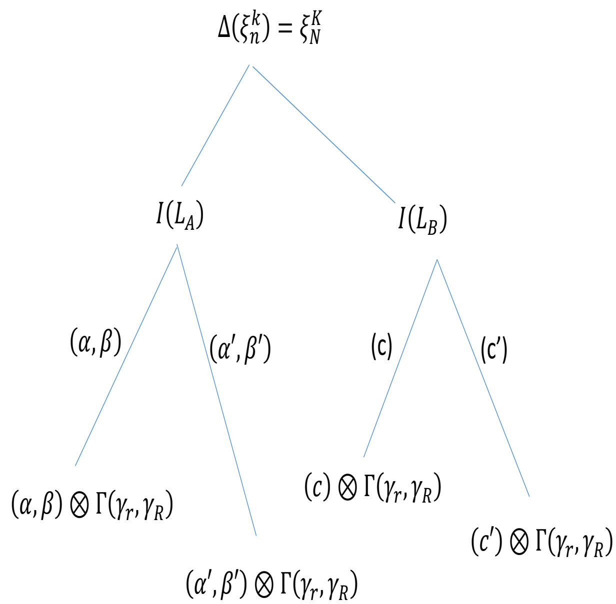


Figure 4-. A dialectic process

In Figure (4-), $(\Delta(\xi_n^k))$ represents a transformed syntax. Two types of interpretations are possible. If syntax is of type 1, called group $(L_A,)$, then $(I(L_A))$, is considered as the interpretation of the transformed syntax that introduces doubts or induces contradictory arguments around the concept of a syntax, then the actions identified in the syntax become probable actions, where the probabilities are denoted as (α,β). Thus one branch applies the probabilities to the action identified by the transformed syntax, $((\alpha,\beta)\otimes\Gamma(\gamma_r,\gamma_R))$. Due to the nature of dialectic other probabilities can be considered. These alternative probabilities are denoted by $(\alpha^{'},\beta^{'})$. The actions can be interpreted as actions with alternative probabilities of occurrence given as $((\alpha^{'},\beta^{'})\otimes\Gamma(\gamma_r,\gamma_R))$. The other branch, presents the interpretation of type 2 syntax , called group $((L_B))$, denoted as $(I(L_B))$. $(I(L_B))$ is transformation that causes dialectic effects. $((L_B))$ is a basic logic syntax, such that the logic is clear and leads directly to action. Thus can cause repetitions of actions denoted as real numbers (c), and (c'). Action in this case is modified by the repetition numbers, $(c\otimes\Gamma(\gamma_r,\gamma_R))$, and $(c'\otimes\Gamma(\gamma_r,\gamma_R))$. An overview of the analysis of language is given in Figure 4.

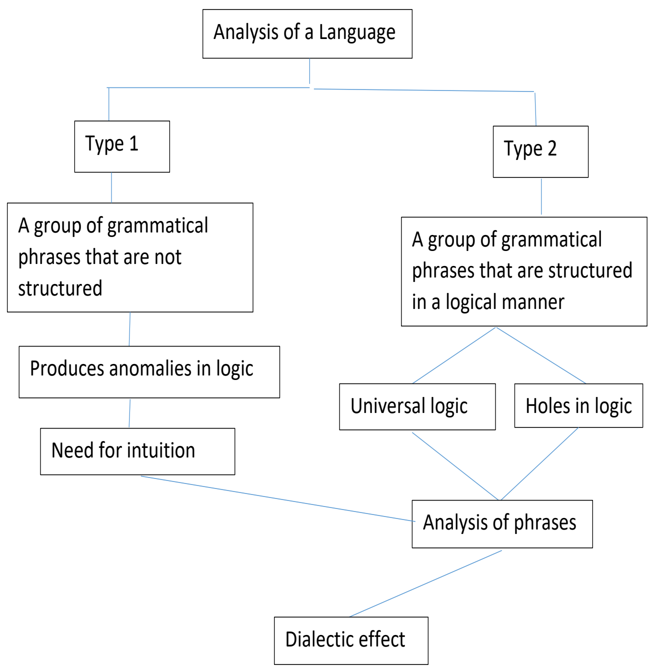


Figure 4. Overview of the analysis of a language

In Figure 4, the analysis of a language consists of distinguishing between (2) types of phrases. Type 1 phrases consist of a group(A) of phrases, denoted as (L_A) that are not strictly structured; thus there is room for interpretation which can cause anomalies in reasoning. In order to justify one type of reasoning over another, it is necessary to use intuition to complete the gaps in the reasoning. Intuition must be an addition to the existing syntax or its' modifications. Generally the gaps in the logic can be completed using syntax operators that allow for additions,(ξ, Δ). Type 2 phrases are group(B) phrases, denoted as (L_B) that consist of phrases that are well structured and therefore, have a certain evident logic. In this case no interpretation or intuition is required. It is always possible to evolve both (L_A) , and (L_B) using what is called here, phrase operators, (Δ). As is shown in Figure 4, syntax emerges from the analysis of a language. The dialectic effect refers to the dynamics of the language in the sense that any phrase in a language allows for a certain number of interpretations. For example a phrase can be taken literally based on the elements (objects, predicates, degrees), involved in the phrase, or it can be interpreted in a positive or negative way depending on the intelligence level, the experience, and the mental and emotional state of an individual. A language is a conceptual space of an infinite combinations and permutations of phrases made out of smaller elements. This is shown as a diagram in Figure 5.

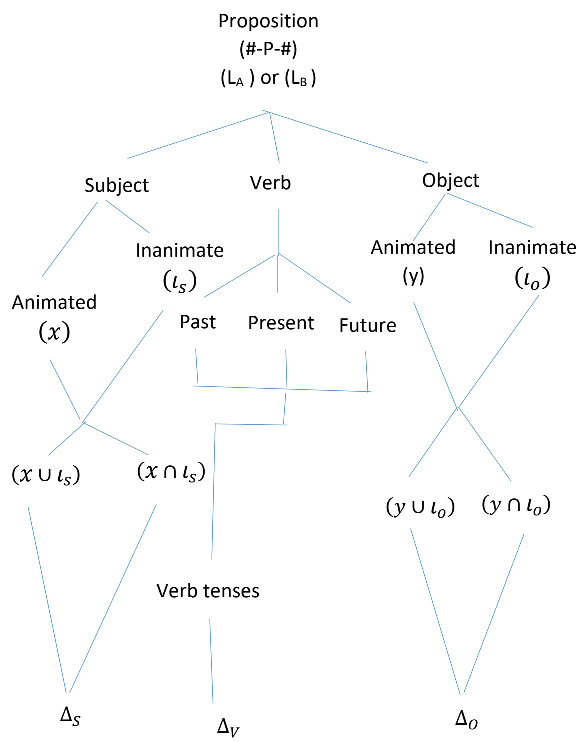


Figure 5. Overview of syntax

As is shown in Figure 5, syntax starts with a basic phrase represented by ($\xi = L_A$), or ($\xi = L_B$), or ($\xi = \sharp - P - \sharp$). In ($\sharp - P - \sharp$), (P) refers to the main phrase, and the sign ($\sharp$) stands for prefixes or post fixes that can be added to the basic phrase (P), which are the elements that construct a phrase. The elements of a syntax are the usual subject, verb, and object.(2) types of subjects are animated, denoted by (x), and inanimate represented as (ι_s). Subject can be constructed as the union of (x), and (ι_s), ($x \cup \iota_s$), or subject is constructed as an enhancement through analogies with (ι_s), presented as an intersection ($x \cap \iota_s$). Verb possess (3) tenses (past, present, future). Similarly, objects can be animate, represented by (y), and inanimate represented as (ι_o). Object can be developed by ($y \cup \iota_o$), or enhanced by ($y \cap \iota_o$). All (3) elements of a phrase, subject, verb and object are considered as open causal sets. Open causal sets are defined as sets with boundary elements that could fall in the neighborhoods of two distinct conceptual sets. Each causal set consists of several open subsets. Open subsets refer to the same concept except that the boundaries of the subsets of the two distinct sets could share common elements. Many combinations, and permutations of subsets in each causal set is possible through the application of operators denoted by, (Δ) which is referred to as syntax transformation operator. In Figure 5, each segment of a phrase, ($\sharp - P - \sharp$), subject, verb, and object, can be transformed through the application of operators that allow for combinations and permutations.

Each segment operators are denoted by $(\Delta_S, \Delta_V, \Delta_O)$. where (S) stands for subject, (V) for verb, and (O) for object. The existence of (Δ), signifies that the original phrase, $(\sharp - P - \sharp)$ is either not complete, or can be modified. The original $(\sharp - P - \sharp)$ is a physical representation of a mental process. A mental process is a process that has an outward manifestation. By this it is meant that each $(\sharp - P - \sharp)$ causes a certain type of action. A more elaborate version of Figure 5, is given in Figure 6.

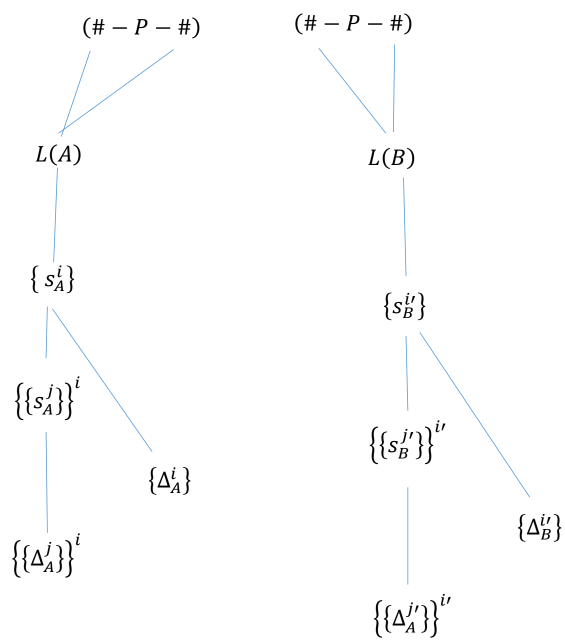


Figure 6. Syntax derived causal sets

Figure 6, demonstrates the formation of causal sets. In Figure 6, a basic phrase, $(\sharp - P - \sharp)$ contains two basic groups of syntax components, here they are named groups (A), and (B). Each syntax group contain causal sets. Causal sets are conceptual sets. A conceptual set consists of objects that each can own specific meaning that can be used along with objects of other causal sets in a phrase to form a logical shape leading to a certain type of action. The following notation is used to describe Figure 6. group (A) is denoted as (L_A). (L_A) has sub-groups denoted by $(L_A = \{s_A^i(X_1, \cdots, X_n)\} = (\xi_{A(n)}^k)^i; \forall i = 1, \cdots, I; k = 1, \cdots, m_n)$, where there are n objects, and (k) predicates shown as a syntax with (k) predicate and (n) objects, $((\xi_{A(n)}^k)^i)$, and (i) represents the sub-group (i). Each sub-group $(\{s_A^i\})$ contains its own sub-groups, $(\{s_A^i\} = (\{s_A^j\})^i = ((\xi_{A(n)}^k)^j)^i; \forall i = 1, \cdots, I, j = 1, \cdots, J; k = 1, \cdots, m_n)$. Similar notation can be used for phrases that contain group (B) syntax segments. Group (B) is denoted as (L_B). (L_B) has sub-groups denoted by

$(L_B = \{s^{i'}_B(X_1, \cdots, X_n)\} = (\xi^k_{B(n)})^{i'}; \forall i' = 1, \cdots, I'; k = 1, \cdots, m_n))$. Each sub-group $(\{s^{i'}_B\})$ contains its own sub-groups,$(\{s^{i'}_B\} = (\{\{s^{j'}_A\}\})^{i'} = ((\xi^k_{B(n)})^{j'})^{i'}; \forall i' = 1, \cdots, I', j' = 1, \cdots, J'; k = 1, \cdots, m_n)$. All sets, and their subsets represent $(\sharp)$ attachment that either enhances (P), in a way that a fixed logic prevails, or induce uncertainty in (P). Thus depending on whether the attachment $(\sharp \in L_A)$ is in group (L_A) or$(\sharp \in L_B)$ in group (L_B). The (2) types of syntax construction has consequences on the type and the quantity of actions that are produced.

The permutation and the combination of the objects, $((X_1, \cdots, X_n))$ is done using operators for sub-groups of group (A) denoted by sets $(\{\Delta^i_A\})$, and for sub-groups of group (B) by $(\{\Delta^{i'}_B\})$. Operators can be applied to the sub-groups of the sub-groups, denoted respectively by $((\{\Delta^j_A\})^i)$, and $((\{\Delta^{j'}_A\})^{i'})$. If the application of operators are needed, then to convey a logic within a syntax, the following application is used. Let the set of objects $((Y_1, \cdots, Y_\upsilon); \upsilon = 1, \cdots, \Upsilon)$ be either from $(L_A \cup L_B)$, where $(\Upsilon > n)$, or $(L_A \cap L_B)$, where $(\Upsilon < n)$ or is chosen from (L_A) or (L_B) where $(\Upsilon = n)$. The application of an operator to sub-groups is represented as $(\{\Delta^i\} = (\Delta)^i(\xi^k_{A(n)})^i = (X_i) \wedge \{Y_\upsilon\}; X_1 \geq X_i \leq X_n; \upsilon = 1, \cdots, \Upsilon)$, and the operators for the sub-groups $((\{s^j\})^i)$ is expressed as $((\{\Delta^j\})^i = (\Delta)^j(\xi^k_{A(n)})^i = (X_j) \wedge \{Y_\upsilon\}; X_1 \leq X_j \leq X_i; \upsilon = 1, \cdots, \Upsilon)$. The same formulation of operators exist for sub-groups $(\{s^{i'}_B\})$, and $(\{s^{j'}_A\})$. An example of an operator for a sub-group $(\{s^i_{A(n)}\})$ is given for (i=1), predicate (k=1), as $(\{\Delta^1\} = (\Delta)^1(\xi^1_{A(n)})^1 = (X_1) \wedge \{Y_\upsilon\}; \upsilon = 1, \cdots, \Upsilon)$. The final choice of the objects depends on the logic or the dialectical logic produced in a syntax.

Definition 2.1. Causal sets are conceptual sets that when put together they take a certain form; they convey a certain logic that as a consequence leads to a certain type of action. By conceptual, it is meant the following conditions:

1. Each object of a conceptual causal set has a simple geometrical meaning by itself. An example of a simple geometrical meaning is a point identified by its coordinates, or a line identified by two separate points. More developed geometrical meanings are triangles, rectangles, squares, and fractals, are a few examples. A more developed geometrical meaning is represented as a collection of points and lines in a hyper space.

2. No two objects within the same conceptual causal set convey the same geometrical meaning. For example they represent two distinct points or two distinct lines with different trajectories, and inclinations, namely vectors, and tensors in a multi-dimensional space.

3. No two conceptual causal sets have any geometrical meaning in common.

4. A conceptual causal set is a null set, if it contains no objects, or if each object in the conceptual set does not have a geometrical meaning.

Causal set operations are the union, and intersection operations. In general the causal sets and their sub-sets do not intersect. This applies to both groups A and B. For group A, $(s^i_A \cap s^{i*}_A) = \varnothing\ \forall i \neq i*)$ while the union is a non-empty set, $(s^i_A \cup s^{i*}_A) \neq \varnothing\ \forall i \neq i*)$. The same applies to sets in group B, $(s^j_B \cap s^{j*}_B) = \varnothing\ \forall j \neq j*)$ while the union is a non-empty set, $(s^j_B \cup s^{j*}_B) \neq \varnothing\ \forall j \neq j*)$. The intersection and the union definitions are considered as conditions (1), and (2) respectively. Based on condition (1), the following relation exists: $(s^i_A - s^{i*}_A = 0 \forall i \neq i*$. (0) represents an empty set. In the case of conceptual causal sets, zero sets imply phrases that do not contain a coherent logic, in other words, the phrase has no meaning. Following the (2) conditions imposed on conceptual causal sets, the following relation among sets exist $(s^i_A \bigtriangleup s^{i*}_A = (s^i_A \cup s^{i*}_A) - (s^i_A \cap s^{i*}_A) = (s^i_A \cup s^{i*}_A) - 0 = (s^i_A \cup s^{i*}_A))$. $(\bigtriangleup)$ represents the disjunctive union or symmetric difference. The following Theorem states that $((s^i_A \cup s^{i*}_A) \neq (s^{i*}_A \cup s^i_A))$.

Theorem 2.1. *The union of any two sets (s^i_A) and (s^{i*}_A), where $(i \neq i*)$ is directional, therefore $((s^i_A \cup s^{i*}_A) \neq (s^{i*}_A \cup s^i_A))$.*

Proof. Given the definition of the disjunctive union, $(s^i_A \bigtriangleup s^{i*}_A = (s^i_A \cup s^{i*}_A), \forall i \neq i*)$. Every union of causal sets constitutes a phrase, thus the union represents a certain geometrical form. When the order of the union is reversed, $(s^{i*}_A \bigtriangleup s^i_A = (s^{i*}_A \cup s^i_A), \forall i \neq i*)$, the modification caused by reversing the order of the union, modifies the logic of the syntax. It either causes change in the basic logic, or create dialectical logic, which in both cases creates unique geometrical shape distinguished from the shape produced by $(s^i_A \cup s^{i*}_A)$, thus the following is true, $((s^i_A \bigtriangleup s^{i*}_A) \neq (s^{i*}_A \bigtriangleup s^i_A))$. □

The causal set relation follows. Causal sets are non-ordered in the sense that the order within the set is insignificant since each element of a set is independent of the other members, and each element conveys a meaning and a specific affine shape. On the other hand the order in combination with other causal sets becomes significant as much as each union of causal sets in the order that is presented conveys a different meaning. An example of a non-ordered causal set is given as $(s = the, a, an, then, \cdots)$. Let another non-ordered set be $(s* = (a, the, these, an, those, this, \cdots))$. Let a subset of (s) be denoted by $(\mathbf{a})$ such that $(\mathbf{a} = (\mathbf{a}) \in s)$, and $(\mathbf{a} = (the, a))$,contains (2) elements. Let a subset of $(s*)$ be denoted as $(\mathbf{b} = (\mathbf{b}) \in s*)$, such that $((\mathbf{b}) = (the, these, an))$, contains (3) elements. Let $((\mathbf{a}, \mathbf{b}) \in (s \times s*))$, and $((\mathbf{a}, \mathbf{b}) = (\mathbf{a}) \times (\mathbf{b}))$. $((\mathbf{a}, \mathbf{b}))$ is called a causal conceptual set. Schematically, different enumerations of $((\mathbf{a}, \mathbf{b}))$ are the following.

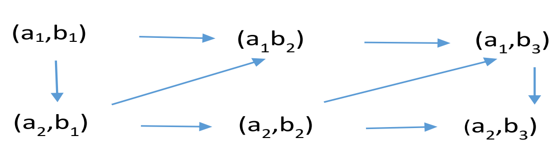


Figure 7. Causal conceptual sets enumerations

In Figure 7, $((\mathbf{a}) = (a_1, a_2))$, represents each element of the set $((\mathbf{a}))$, and $((\mathbf{b}) = (b_1, b_2, b_3))$, represents each element of set $((\mathbf{b}))$. The arrows represent conceptual equivalency of each subset to the others. The order becomes significant when the two sets $((\mathbf{a})$, and $((\mathbf{b}))$ are in combination with the third set denoted as $((\mathbf{c}))$, where together with the set $((\mathbf{c}))$, $((\mathbf{a}), (\mathbf{b}), (\mathbf{c}))$ make up a phrase. The reason is that each specific combination of the three sets is physically represented by a set of movements or actions that geometrically are shown by a different sets of lines representing different planes. Given sets $((\mathbf{a}) = (a_1, a_2, \cdots, a_n)$,$((\mathbf{b}) = (b_1, b_2, \cdots, b_m)$, and $((\mathbf{c}) = (c_1, c_2, \cdots, c_N)$, then $((a_i), (b_j), (c_k) \neq (a_i), (b_j), (c_k); i \neq j \neq k; \forall i = 1, \cdots, n; j = 1, \cdots, m; k = 1, \cdots, N)$. An example of the correspondence between a phrase, represented by $((a_i), (b_j), (c_k))$ and sets of movements or actions represented by a plane in 2D is shown below in Figure 8.

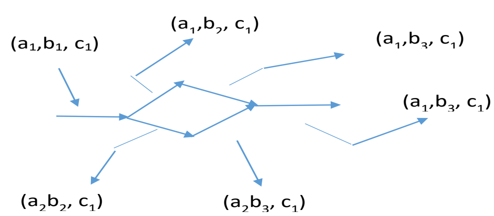


Figure 8. Syntax (ξ, Δ) represented as a manifold

In Figure 8, the set (c) is fixed, $((c) = c_1)$. Let $((a) = (a_1, a_2)$, and $((b) = b_1, b_2, b_3)$. The plane shown in Figure 8, is a hypothetical representation of the different combination of the sets (a), and (b), with a fixed set (c), $((a_i), (b_j), c_1)$. As can be seen each line represent a particular combination and thus a certain action corresponding to it. The details of action are discussed in the section on action. Using the example introduced earlier, if the combination of the sets changes from $((a_i), (b_j), (c_1); i = 1, 2; j = 1, 2, 3)$ to $((a_i), c_1, (b_j),)$, then a different manifold is obtained that represents a corresponding collection of actions. Given Figure 8, as an example, the manifold in 2D of the set $((a_i), c_1, (b_j),)$, is shown in Figure 9.

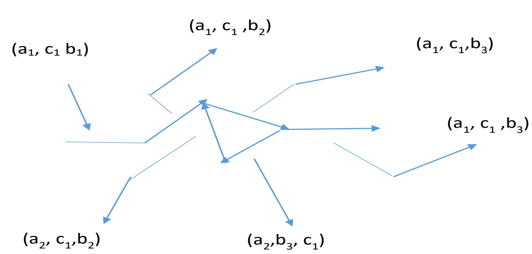


Figure 9. An alternate manifold representing a changed combination of sets

In Figure 9, the combination $((a_i), c_1, (b_j),)$ produces a new manifold. Though the new actions are represented by the same lines in the conceptual space, (ξ, Δ) , it is the characteristics of the lines that are altered. These characteristics, are the angle of rotation, the direction of the lines, and the magnitude of lines. The modified conceptual lines with new characteristics are mapped onto the movements or actions described in the section on action. In general, every conceptual set is a countable set. There is a finite number of elements in each conceptual set. The union of conceptual sets are countable. Thus the set of actions produced by the union of conceptual sets is countable. There is a finite collection of actions. In other words, the number of possible actions for each union of conceptual sets is fixed due to inherent nature of each set. The two Figures convey two seperate paths dictated by specific actions leading to two different outcomes. Since each branch indicates a specific action, thus different combination of actions produce different end results. This means different demand and supply characteristics in the context of Dialectic Causal Theory of Action (DTCA).

Theorem 2.2. *Given subsets* $((\mathbf{a}) \in s)$,$((\mathbf{b}) \in s^*)$, *then* $((\mathbf{a}), (\mathbf{b})) = ((\mathbf{b}), (\mathbf{a}))$ *iff* $(\mathbf{a}) = (\mathbf{b})$, *otherwise,* $((\mathbf{a}), (\mathbf{b})) \neq ((\mathbf{b}), (\mathbf{a}))$.

Proof. Let sets $(\mathbf{a}) = (\mathbf{b})$, then $((\mathbf{a}) \cap (\mathbf{b})) = (\mathbf{a})$, and $((\mathbf{b}) \cap (\mathbf{a})) = (\mathbf{a})$, the two sets contain the same elements. Let $(\mathbf{a}) \neq (\mathbf{b})$, then $((\mathbf{a}) \cap (\mathbf{b})) = 0$, and therefore, $((\mathbf{a}), (\mathbf{b})) \neq ((\mathbf{b}), (\mathbf{a}))$. □

Theorem 2.2 can be extended to any subset of the space of the causal sets (ξ, Δ). This is presented as a Lemma below.

lemma. *Given subsets* $((a_l) \in s^i; l = 1, \cdots, L)$, *and* $((a_{l^*}) \in s^{i'}; l' = 1, \cdots, L^*)$, *then* $((a_l), (a_{l^*}) = (a_{l*}), (a_l))$ *iff* $((a_l) = (a_{l^*}))$.

It is shown that the union of any two sets are ordered. This is generalized to the union of several causal sets as is shown in lemma 2. Thus different combinations, and permutations of sets produce different conceptual hyper-planes where in the homeomorphic mapping translates to the 3 dimensional Euclidean space representing action. The combination and permutation (union) of different subsets of different groups produces unique conceptual hyper-planes. Based on Figure 8, an example of various hyper planes are given in Figure (8+). The explanation is formally given in Theorem 2.3. In Figure 8, the hyper-plane is constructed with (6), tensors. This construction is due to a syntax that contains let's say (n) objects, and (6) predicates, giving the syntax a degree equal to (6). Figure 9, demonstrates the change in a conceptual manifold, due to different combination of the union of subsets. Now if, there exists another subset not any of the (2) subsets identified, then the union of the sets, $(\mathbf{a})$, and $(\mathbf{b})$ with the new subset creates new tensor manifolds that are the modification of the basic manifold, Figure 8. For example, in Figure (8+), the tensor manifold is modified by adding (4) more predicates giving the hyper-plane twists of an upward, and down ward directions. In this case, the dimension of the manifold, i.e. is changed from (6) to (10). In addition, there are two twists, caused by the change of angle, causing change of direction. The extra lines augment the length of the causal manifold. The formal argument is given in theorem 2.3.

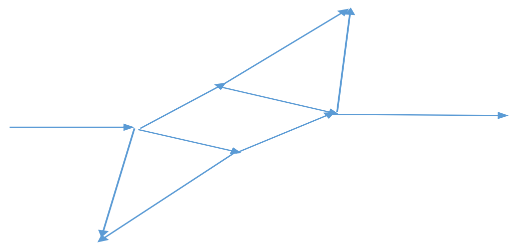


Figure 8+. Syntax (ξ, Δ) modification represented as an unique manifold

Theorem 2.3. *Let set $((\mathbf{a}) \in s_A)$, and let $((\mathbf{b}) \in s_B)$, where (A) is the space of all groups groups (L_A), and $(s_A \subset L_A)$ is a categorical subset of group (L_A). $(s_B \subset L_B)$, is a categorical subset of group (L_B) in the causal conceptual space of groups (L_B), and $((\mathbf{b}) \in s_B)$, such that $((\mathbf{a}) \cap (\mathbf{b}) = 0)$, and $((\mathbf{a}) \cup (\mathbf{b}) \neq 0)$. Both groups (L_A), and (L_B) are in the syntax , $(L_A \in \xi)$, and $(L_B \in \xi)$. Both sub-groups relate the same number of predicates and degree. This implies that the sum of sets $((\mathbf{a}))$, and $((\mathbf{b}))$ make up (n) number of objects, $(X_1, \cdots, X_n)$, and $(k; k = 1, 2, \cdots, m_n)$ predicates , and thus the same degree. Let set $(((\mathbf{c}))$ be a subset of (s_C), $((\mathbf{c}) \in s_C)$, where $(s_C \not\subset s_A)$, and $(s_C \not\subset s_A)$, and$(s_C \not\subset s_B)$. Thus (s_C) belong to the space (C) of all groups (L_C). The set $(s_C \in \Delta)$. (Δ) is a piece of syntax with $(m \neq n)$ objects different from (ξ), and $(k^*; k^* = 1, 2, \cdots, m_m)$ predicates , and thus the same degree, $(\mathfrak{d} = k^*)$. Then any combinations and permutations, $((\mathbf{a}), (\mathbf{b}) \wedge (\mathbf{c}))$, produce a distinct conceptual hyper-planes denoted by $(\mathbb{H}_h(\xi, \Delta); h = 1, 2, \cdots, H)$, (h) represents the number of hyper-planes. No two hyper-planes are the same, $(\mathbb{H}_h(\xi, \Delta) \neq \mathbb{H}_{h'}(\xi, \Delta))$.*

Proof. The outline of the proof is to show that any modifications of a syntax, (ξ), by adding syntax transforms (Δ), creates distinct conceptual hyper-planes. The reason is due to 1), change in the dimension of each hyper-plane; 2) change of angle of rotation from the initial syntax, (ξ). 3) elongation or shrinkage of the initial syntax, (ξ). Let's denote a causal conceptual hyper-plane as $(\mathbb{H}(\xi, \Delta))$. Each hyper-plane is constructed out of a collection of tensors derived from the objects, $(X_i = (X_1, \cdots, X_n); i = 1, 2, \cdots, n)$, predicates, $(X_k; k = 1, 2, \cdots, m_n)$, where $(1 \geq k \leq n)$, $(X_k \subset X_i)$, and degree, $(\mathfrak{d} = k)$. Let a hyper-plane $(\mathbb{H}(\xi, \Delta))$, be constructed from $(X_i; i = 1, 2, \cdots, n)$ objects, $(X_k; k = 1, 2, \cdots, m_n)$, predicates, and $(\mathfrak{d} = k)$ degree. Now let the (n) objects belong to two conceptual groups (L_A), and (L_B). Thus, some objects belong to one group or another. Let $((\mathbf{a}) \in s_A)$,and $((\mathbf{b}) \in s_B)$. The tensor representation of $(\mathbb{H}(\xi, \Delta))$ is formulated as $(\mathbb{H} = \sum_i \sum_k (\bar{a})_i^k X_k)$, where $(\bar{a} = (a + b) \times cosh(\theta_{X_k, X_{k'}}); k \neq k')$, and $((\bar{a})_i^k; i = 1, 2, \cdots, n; k = k = 1, 2, \cdots, m_n)$ is a matrix of size $(n \times k)$. $(cosh(\theta_{X_k, X_{k'}}))$ represents the angle between any two different reference frames. Conceptual Hyper-planes exist in a non-Euclidean space, where the predicates in a syntax constitute the frame of reference. Unlike the normal Euclidean space, the coordinates that constitute the space are not orthogonal to each other. They stand at an angle from each other, and thus, possess a covariance. The covariance is a function of the angle of rotation. $((\bar{a})_i^k)$ is coefficient matrix representing the change of the reference frame (X_k).

Each element $((\bar{a})_i^k)$, is the magnitude of the difference between the object and the predicate, $((\bar{a})_i^k = |(\bar{a})_i - (\bar{a})_k|)$. Both subsets $((\mathbf{a}))$, and $((\mathbf{b}))$ share the same reference frame, i.e. the same predicates, (X_k). Now let (Δ) be a syntax transformation such that the syntax (ξ) is changed in the following way, $(((\mathbf{a}) \cup (\mathbf{b})) \wedge (\mathbf{c}))$. The hyper-plane resulting from (ξ, Δ) has predicates $(X_k \oplus X_{k^*}^c)$. The tensor representation of the resulting hyper-plane $(\mathbb{H}_{k,k^*})$ is formulated as follows four tensor blocks $(\mathbb{H}_{k,k^*} = \Lambda_{\bar{a}}; \mathbf{0}_1; \mathbf{0}_2\ ; \Lambda_{\bar{c}} \otimes (X_k, X_{k^*}))$, where $(\Lambda_{\bar{a}} = \sum_i \sum_k (\bar{a})_i^k)$ is an $(n \times k)$ block, and $(\mathbf{0}_1)$ is an $(n \times k^*)$ block with all elements zero. $(\mathbf{0}_2)$ is an $(m \times k)$ block with all elements zero. $(\Lambda_{\bar{c}})$ is an $(m \times k^*)$ block matrix, with each element of the matrix being (c_{i^*,j^*}) multiplied by a $(k \times k^*)$ matrix of angles $(\phi_{i',j'}; i' = 1, 2, \cdots, k; j' = 1, 2, \cdots, k^*)$. The angle $(\phi_{i',j'})$ is the angle between the two hyper-planes$(\mathbb{H})$, and the hyper-plane made up of the tensors $(\Lambda_{\bar{c}})$.

□

The following Theorem proves that there exists a countable number of actions produced by each combination, (union, intersection) of causal conceptual sets.

Theorem 2.4. *Given subsets $((\mathbf{a}) \in s_A^i)$,$((\mathbf{ab}) \in s_A^{i'})$, such that $(s_A^i \subset A)$, and $(s_A^{i'} \subset A)$, $(i \neq i')$. (A) is the space of all causal sets of group (L_A); then any combination, $(\cup, \cap)$ of the two sets $((\mathbf{a}))$, and $((\mathbf{b}))$, $(((a), (b)) = (s_A^i \times s_A^{i'}))$, where $(((\mathbf{a}), (\mathbf{b})) = ((\mathbf{a}) \cap (\mathbf{b})))$, or $(((\mathbf{a}), (\mathbf{b})) = ((\mathbf{a}) \cup (\mathbf{a})))$ is countable.*

Proof. $((\mathbf{a}))$ is a countable causal set, thus it can be arranged either in a finite or infinite sequence as $((\mathbf{a}) = (a_1, a_2, \cdots)$or $((\mathbf{a}) = (a_1, a_n, a_2, \cdots)$, where (n) is the total possible number of elements in the causal set $((\mathbf{a}))$. Let the causal conceptual set $((\mathbf{b}))$ be a fixed causal set such that $((\mathbf{b}) = b^-)$ with one element in the causal set $((\mathbf{b}))$. Consider $(((\mathbf{a}), (\mathbf{b})) = ((\mathbf{a}), (b^-)))$ to represent either a union or an intersection of the two causal sets $((\mathbf{a}))$, and $((\mathbf{b}))$. Since $((\mathbf{b}) = b^-)$ and the causal set $((\mathbf{a}))$ are countable sets, then $(((\mathbf{a}), (\mathbf{b})))$ is countable. Let $((\mathbf{b}))$ be a fixed set, then for each element of the causal set $((\mathbf{b}))$ there exists a countable combination $(((\mathbf{a}), (\mathbf{b})))$, that constitutes a countable family of countable causal conceptual sets since each of the combinations of the causal sets is countable. □

Theorem 2.4 can be generalized to include several causal conceptual sets. The following Lemma states a generalization of the combination of causal sets $((\mathbf{a}), (\mathbf{b})))$ with other causal sets that remain countable.

Lemma 2.5. *Given that the combination $(\cup, \cap)$ causal conceptual set $(((\mathbf{a}), (\mathbf{b})))$ is countable, and a fixed causal conceptual set $((\mathbf{c}) \notin s_A)$, and $((\mathbf{c}) \notin s_B)$, where $(s_A \subset A)$, and $(s_B \subset B)$. $((\mathbf{c}) \in s_C)$, $(C \neq A)$, where (C) is the space of conceptual causal sets not in space (A), and $(C \neq B)$, (representing other conceptual causal sets in groups other than group (L_A), and group (L_B), then the combination causal conceptual set, $((\mathbf{a}, \mathbf{b}, \mathbf{c}))$ is countable.*

Proof. The proof is similar to Theorem 2.4. The combination causal conceptual set $((\mathbf{a}, \mathbf{b}, \mathbf{c}))$ is countable, using the same reasoning any conceptual causal set either from the causal space of groups (A),(L_A) or any other conceptual causal group with countable number of elements can be added by taking a fixed element of the causal set ($\mathbf{c}$), as follows: let $((\mathbf{a}, \mathbf{b}, \mathbf{c}) = \aleph^n; \mathbf{c} = 0)$, where ($\aleph$) represents a countable set, with at most (n) elements, given that ($\mathbf{c} = 0$) is a set with zero elements. Then any combination causal conceptual set with causal set ($\mathbf{c}$)) can be constructed as ($\aleph^{n+1} = \aleph^n \cup c_1$), given that the set $(\mathbf{c}) = c_j; j = 1, 2, \cdots, m)$ is countable, and while ignoring any union with countable sets with negative elements, is considered as an empty set, $((\mathbf{a}, \mathbf{b}) \cup (\mathbf{c}_-) = \mathbf{0}$), where $((\mathbf{c}_-))$ represent a countable set of negative numbers. The combination conceptual causal set ($\aleph^{n+1}$) is countable , and any consequent combination conceptual causal set ($\aleph^{n+2} = \aleph^n + \aleph^{n+1} + c_2$), therefore, ($\aleph^{n+2}$) is countable. Expanding the same reasoning one can conclude that any extension of a countable set is countable, ($\aleph^{n+j} = \aleph^n + \aleph^{n+1} + c_j; j = 3, \cdots, m$). The union of all the combination conceptual causal sets ($\cup_{k=n}^{n+m} \aleph^k$) is countable. □

The ordering issue of causal conceptual sets can be extended to the multiplicative case. Let (A) be a space of syntax groups of type (L_A), and let (B) the space of syntax groups of type (L_B). Each group (L_A), and (L_B) contains categories designated by ($s_A^i; i = 1, 2, \cdots, N$), where (i) is the number of subcategories, and (N) is the maximum number of sub-categories. The same for group (L_B), ($s_B^j; j = 1, 2, \cdots, M$), where (j) is the number of subcategories in group (L_B), and (M) is the maximum number of categories in group (L_B). The ordering of sub categories can be demonstrated. Let ($\mathbf{a}_i = s_A^i$) , and ($\mathbf{b}_{i'} = s_A^{i'}$). Let $(((\mathbf{a}_i), (\mathbf{b}_{i'})) = (s_A^i \times s_A^{i'}); \forall i \neq i')$. Let $((\mathbf{a}_i^k) = s_A^{i,k}; s_A^{i,k} \in s_A^i$), where ($s_A^{i,k}$) is a sub-category with (k) elements. Let $((\mathbf{a}_{i'}^{k'}) = s_A^{i',k'}; s_A^{i',k'} \in s_A^{i'}$), where ($s_A^{i',k'}$) is a sub-category of group (i') with (k') elements. Then $(((\mathbf{a}_i^k), (\mathbf{b}_{i'}^{k'})) = (s_A^{i,k} \times s_A^{i',k'}); \forall i \neq i')$. $(((\mathbf{a}_i^k), (\mathbf{b}_{i'}^{k'}))$ is called a causal conceptual product of subcategories. As in the case of addition, the two conditions are maintained; $(((\mathbf{a}_i), (\mathbf{b}_{i'})) = ((\mathbf{b}_{i'}), (\mathbf{a}_i))$ iff $((\mathbf{a}_i) = (\mathbf{b}_{i'}))$, otherwise, $(((\mathbf{a}_i), (\mathbf{b}_{i'})) \neq ((\mathbf{b}_{i'}), (\mathbf{a}_i)$. The same condition applies to the subcategories. The causal conceptual product can be extended to include more categories and subcategories,$(((\mathbf{a}_i), (\mathbf{b}_{i'}), (\mathbf{c}_{i"})) = (s_A^i \times s_A^{i'} \times s_A^{i"}); \forall i \neq i' \neq i")$ where $((\mathbf{c}_{i"}) = s_A^{i"}$). The same is true for the subcategories, $(((\mathbf{a}_i^k), (\mathbf{b}_{i'}^{k'}), (\mathbf{c^{k"}}_{i"})) = (s_A^{i,k} \times s_A^{i',k'} \times s_A^{i",k"}); \forall i \neq i' \neq i")$. The causal conceptual product has the following property: $(((\mathbf{a}_i), (\mathbf{b}_{i'}), (\mathbf{c}_{i"})) = ((\mathbf{a}_i), (\mathbf{c}_{i"}), (\mathbf{b}_{i'})) = ((\mathbf{b}_{i'}), (\mathbf{c}_{i"}), (\mathbf{a}_i)) = ((\mathbf{b}_{i'}), (\mathbf{a}_i), (\mathbf{c}_{i"})) = ((\mathbf{c}_{i"}), (\mathbf{a}_i), (\mathbf{b}_{i'})) = ((\mathbf{c}_{i"}), (\mathbf{b}_{i'}), (\mathbf{a}_i)))$ iff $((\mathbf{a}_i) = (\mathbf{b}_{i'}) = (\mathbf{c}_{"i}))$. The same condition is true for subcategories.

Dialectic reasoning is the inherent part of a syntax of a phrase. This means that syntax has to have in general a certain sense of coherence. Here it is assumed that a syntax possesses one of the following types of reasoning: 1) an embodied meaning; 2) a logical approach; 3) a reasoning technique. Thus the structure of a syntax should convey one of the above mentioned structures.

An embodied meaning refers to a chain of reasoning. Let's take the basic phrase ($\sharp - p - \sharp$) owning a syntax to convey a meaning . Let this meaning be denoted by (ξ). Let any transformation of this phrase ($\sharp - p - \sharp$) be denoted by ($T(\sharp - p - \sharp)$) to create an embodied meaning: $(\Delta(\xi) : (\xi) \to \xi(\Gamma(\gamma_r, \gamma_R)))$, where the enhanced meaning in the syntax , (ξ) causes a set of actions $((\Gamma(\gamma_r, \gamma_R))$, and where ($\Gamma$) represents action, and (γ_r, γ_R) is physical movements (work) within zones of small (r), and large (R) zones in accordance with an individuals physical movements, and his interaction with the physical environment. These sets of actions, $((\Gamma(\gamma_r, \gamma_R))$, in the context of this paper lead to an economic act such as consumption,(purchasing a product).

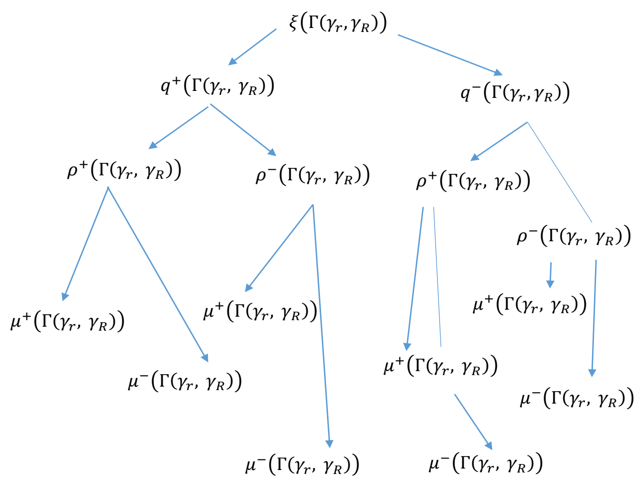


Figure 10. An embodied meaning tree

In Figure 10,$(\xi(\Gamma(\gamma_r, \gamma_R)))$ is the embodied meaning in the syntax. Embodied meaning is a logic that manifests itself in physical movements within an identified environment called action. This means that the meaning induces movement (action) that is the outcome of a cognitive understanding of the physical movements of the body, and its interaction with the physical environment. In the embodied meaning dialectic is an integral part. The dialectic reasoning is shown as positive and negative aspects of the elements involved. These actions in the context of this topic, correspond to economic outcomes mainly they create demand. This demand is denote as $(q(\Gamma(\gamma_r, \gamma_R))$. Demand as a function of action (movement) can be either positive or negative. Positive actions derived from the embodied meaning in the syntax are denoted by $(\xi^+(\Gamma(\gamma_r, \gamma_R)))$. Positive actions lead to positive positive demand $(q^+(\Gamma(\gamma_r, \gamma_R))$. Positive demand,$(q^+(\Gamma(\gamma_r, \gamma_R))$ is the act of purchasing and using a product in various quantities. In general, these are actions that help an individual to consume a certain amount of a product. Negative actions $(\xi^-(\Gamma(\gamma_r, \gamma_R)))$ result in negative demand $(q^-(\Gamma(\gamma_r, \gamma_R))$. Negative demand,$(q^-(\Gamma(\gamma_r, \gamma_R))$ refers to actions that result in either no purchase or purchase below an acceptable level of consumption. In general, negative actions are those type of actions that tend to dissuade an individual towards consumption, or reduce the quantity consumed, denoted by negative demand, $(q^-(\Gamma(\gamma_r, \gamma_R))$. The positive and negative demand are due to the dialectic of action.

In Figure 10, the main derivatives of demand are, consumer surplus, denoted by $(\rho(\Gamma(\gamma_r, \gamma_R))$. Consumer surplus is defined in terms of action, and is the difference between actions taken by an individual to consume and the standard acceptable actions associated with the same consumption. Positive consumption,$(q^+(\Gamma(\gamma_r,\gamma_R))$, means positive consumer surplus, $(\rho^+(\Gamma(\gamma_r,\gamma_R))$. The more an individual consumes the more value is given to the consumption, namely, the actions are perceived to justify consumption. On the other hand, no consumption, or consumption lower than the minimum acceptable, indicates negative consumer surplus, $(\rho^-(\Gamma(\gamma_r,\gamma_R))$. Negative consumer surplus, means that an individual does not value consumption as an activity that has an added value. This is defined as the difference between the activities performed by an individual and the standard activities being quantitatively significant. In general, consumer surplus perceived from the quantity consumed depends on causal action. Positive consumer surplus, produces positive utility, denoted by $(\mu^+(\Gamma(\gamma_r,\gamma_R))$. Utility is defined as a function of action embedded in syntax. Utility in this context refers to relative satisfaction of the actions taken with respect to consumption. Positive utility, refers to positive values given to actions taken. Negative utility, $(\mu^-(\Gamma(\gamma_r,\gamma_R)))$, refers to low or negative values associated to each activity that corresponds to negative consumer surplus,$(\rho^-(\Gamma(\gamma_r,\gamma_R))$. Depending on the consumer surplus, the added value is defined as the acceptable price, $(p(\Gamma(\gamma_r,\gamma_R))$ for a product that depends on causal action. This means that price is determined based on causal action, $((\Gamma(\gamma_r,\gamma_R))$, independent of the quantity consumed. Each activity corresponds to a level of consumption, and a price for that amount of consumption. Each positive or negative consumer surplus, $(\rho^+(\Gamma(\gamma_r,\gamma_R))$, and $(\rho^-(\Gamma(\gamma_r,\gamma_R))$ cause positive, and negative utilities , here designated as $(\mu^+(\Gamma(\gamma_r,\gamma_R))$, and $(\mu^-(\Gamma(\gamma_r,\gamma_R))$. In the case of positive consumer surplus, negative utility diminishes as a consequence of the diminishing returns due to extra consumption. In the case of negative consumer surplus,f positive utility is due to the after thought assessing the consumption as positive. In the embodied meaning approach, the inherent meaning of the syntax, creates actions that define the construction of a demand and supply planes.

The logical approach refers to the syntax that represents the following homology.

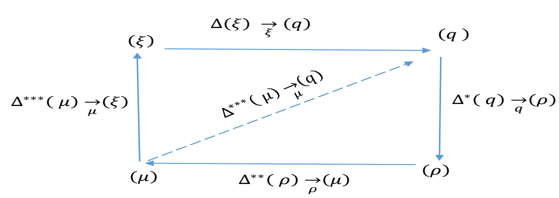


Figure 11. The commutative diagram of logical approach

In the logical approach two elements are ignored. One is the idea that the syntax (ξ) implies a set of actions that lead to demand, (q). In this approach syntax, (ξ) could imply any logical goal such as monetary gains, or alternative profits derived from consumption. Action has no priority over the other logical outcomes of a syntax. The second element is that even if action in syntax is the causal element of the occurrence of demand, the derivatives of demand, (q), the consumer surplus, (ρ), and the utility, (μ) may not relate to the initial syntax driven actions that lead to the demand. The homemorphic functions, $(\Delta, \Delta^{**}, \Delta^{***})$ are projection functions. Thus even if demand is the causality of action, once it happens, then the next projection function to consumer surplus, $(\Delta(\xi) \rightarrow (q))$, can be a function of other elements than the initial causal action. The same argument applies to the projection function (Δ^{*}), where $(\Delta^{*}(q) \rightarrow (\rho))$. The projection function (Δ^{**}), where $(\Delta^{**}(\rho) \rightarrow (\mu))$. The homeomorphic function, (Δ^{***}) is a projection,$(\Delta^{***}(\mu) \rightarrow (\xi))$ that leads to the syntax (ξ) which is the causality of action (Γ) as one possible projection. But in fact another projection function would be $(\Delta^{***}(\mu) \rightarrow (q))$, the one that projects to demand. The dialectic reasoning embedded in syntax plays no role in the logical approach. Demand, (q) does not have any contradictory aspect, and neither do any of its derivatives. Demand, (q) is simply demand. Causal action does not have a dialectic impact on this element. The lack of dialectic impact reduces the efficiency of the logical approach, and reduces it to a standard economic reasoning method which undermines the importance of DCTA economic doctrines.

The reasoning technique approach can be formulated as is shown in the diagram below:

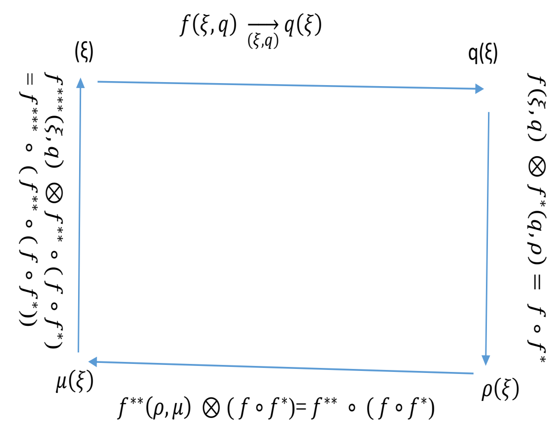


Figure 12. The homeomorphic diagram of reasoning technique approach

In Figure 12, the homemorphic functions are in fact smooth affine manifolds. When the syntax causality is equivalent to a set of actions, $(\Gamma(\gamma_r, \gamma_R))$, then the homemorphic functions become continuous, affine hyperplanes. The smoothness of actions is embedded in the syntax, (ξ). Syntax identifies, 1) the objective, 2) the means to reach the objective. Thus, once the (2) elements are identified, unless a syntax transformation (Δ)introduces reasonable doubts, or holes in the syntax logic, the actions are smooth in the sense that they follow an order of execution. The smooth affine hyperplanes are denoted as $(\mathbb{H}(\Gamma(\gamma_r, \gamma_R)))$. The characteristic of these hyperplanes is that they have Poisson boundaries. A Poisson boundary is denoted as $(\delta = p(\Gamma_k(\gamma_r, \gamma_R)))$, where (δ) is the Poisson boundary, and (p) is the probability of the occurrence of actions of type (k). This characteristic is significant as it allows for a probabilistic approach to the dialectic causality of a syntax. The causal hyperplanes with Poisson boundary property, are considered as coverings. A covering, is used to build a collection of movements from each interpretation of a syntax.

A covering is denoted as $(c_t = (\Gamma_k(\gamma_r, \gamma_R)))$, where (c_t) is a covering which is a set of group (t) actions, $(t = \{k_1\}, \{k_1\}, \cdots, \{k_.\})$. The hyperplane can be written in terms of the coverings as $(\mathbb{H}(\Gamma(\gamma_r, \gamma_R)) = \sum_k c_t \times \delta_k = \sum_k \{(\Gamma_k(\gamma_r, \gamma_R))\} \times p(\Gamma_k(\gamma_r, \gamma_R))$, where (p) is the probability of the occurrence of type (k) actions. Given the nature of a syntax, (ξ), that leads to various collections of movements or actions, the quality of the Poisson boundary is useful in the sense that it considers a large number of movements due to different interpretations of (ξ). The homeomorphic functions $(f, f^*, f^{**}, f^{***})$, as tensor operators, are acting on (ξ), (q), (ρ), and (μ). In order to help the more indepth aspect (dialectic), and allow for a formal mathematical construction of causal syntax, the two methods of the embodied meaning, and the reasoning technique are put together. Thus the existence of the embodied meaning, and the reasoning technique as one entity gives birth to the (DCTA), Dialectic Causal Theory of Action economic system. The dialectic is now transformed into a probabilistic functions and can be measured quantitatively. Figure (12+) demonstrates a summary of the hyperplane presentation using the embodied meaning and the reasoning technique given syntax (ξ), and the operator (f), Poisson boundary, (δ_k), and covering (c_t), . As is shown in Figure (12+) , given the reasoning technique, the operator $(\mathbf{f})$ acts on the syntax (ξ) to create a conceptual manifold, $(\mathbf{f}(\xi))$. Given the structure of the embodied meaning, from the syntax, (ξ), a set of actions, $(\Gamma_k(\gamma_r, \gamma_R)))$ is identified. The identified actions, are identified as being from various sets and subsets of conceptual groups, denoted as (t), $(t = \{k_1\}, \{k_1\}, \cdots, \{k_.\})$. The sets of groups of action constitute coverings, $(c_t = \{\Gamma_k(\gamma_r, \gamma_R))\})$. The Poisson boundary is identified as the probability of the occurrence of the identified actions, $(\delta_k = p(\Gamma_k(\gamma_r, \gamma_R)))$. The hyperplane $(\mathbb{H})$ is a homeomorphism from $(\mathbf{f}(\xi))$ to $(\mathbf{f}(\Gamma_k(\gamma_r, \gamma_R)))$.

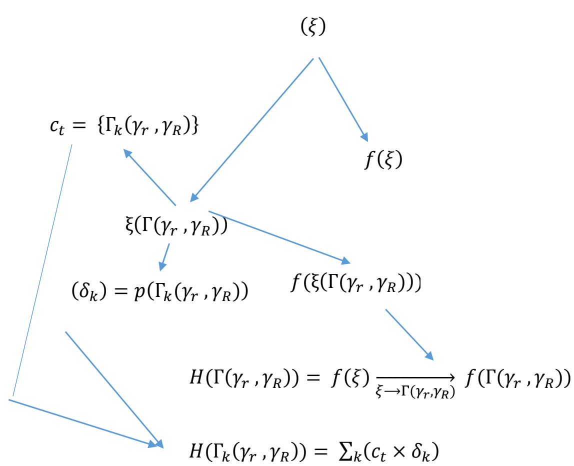


Figure 12+. Hyperplane presentation using the embodied meaning and the reasoning technique

Syntax can be formulated mathematically, as an affine hyper-plane. A phrase represents a syntax. Let a phrase containing a basic syntax, be denoted as (ξ). ($\xi = \{s_A^i, s_B^j\}$) is a causal set consisting of causal conceptual sub-sets; (s_A^i) of the first causal conceptual set ($s_A^i \subset L_A$), ($i = 1, \cdots, n$) is the number of causal conceptual subsets in (L_A), and (s_B^j) of the second causal conceptual set ($s_B^j \subset L_B$), where ($j = 1, \cdots, m$) is the number of causal conceptual subsets in (L_B). The basic phrase (ξ) is represented as a tensor. A group of tensors represent transformations of the basic phrase and thus constitute a tensor hyper plane. A transformation is a modification of the basic phrase due to the dialectic reasoning caused by different perceptions of the syntax of the phrase.

Transformations of the syntax can be obtained by multiplication of extra causal conceptual subsets, (s_A^i), or (s_B^j), in order to achieve the perceptual logic. A syntax (ξ) is written as a tensor: ($\xi = ((\bigcup_{k=1}^{K_A} \bigcup_{i=1}^{n} (P_{K_A}^k)\{s_A^i\}) \oplus (\bigcup_{k'=1}^{K_B} \bigcup_{j=1}^{m} (P_{K_B}^{k'}\{s_B^j\})) \otimes \mathbf{\Delta}$), where ($P_{K_A}^k = \frac{K_A!}{k!(K_A-k)!} \forall K_A \leq k_n$) is a permutation selecting (k) causal conceptual subsets out of a total of (K_A) possible subsets ($K_A \leq k_n$) of causal conceptual subsets ($s_A \subset L_A$), and ($P_{K_B}^{k'} = \frac{K_B!}{k'!(K_B-k')!} \forall K_B \leq k'_m$) is a permutation selecting (k') causal conceptual subsets out of a total of ($K_B \leq k'_m$) possible subsets of causal conceptual subsets ($s_B \subset L_B$).

($\mathbf{\Delta}$) is a tensor operator of transformation based on different permutations of ($s_A^i \subset L_A$, and $s_B^j \subset L_B$). The tensor notation of the segment of the syntax (ξ) before the application of the operator ($\mathbf{\Delta}$) is denoted by ($\bar{\xi}$). The tensor formulation of ($\bar{\xi}$) is given as, ($\bar{\xi} = \sum_k^{K_A} \sum_i^n P^{i,k}(s_A)_i \oplus \sum_{k'}^{K_B} \sum_j^m P^{j,k'}(s_B)_j$). The syntax tensor ($\bar{\xi}$) can be represented in a matrix format as matrix ($\bar{\xi} = \mathbf{A}$) consisting of (4) blocks of matrices (A_1, A_2, A_3, A_4). Block (A_1) is a matrix of size ($n \times K_A$). The elements of (A_1) are ($A_1 = \sum_k^{K_A} \sum_i^n P^{i,k} \times (\mathsf{d})_i$), where ($\mathsf{d})_i$), is the degree of subset (s_A^i). The degree of a subset is the number of predicates of that subset. Block (A_2) is a matrix of size ($n \times K_B$) with all elements zeros ($A_2 = \mathbf{0}$). Block (A_3) is a matrix of size ($m \times K_A$) with all elements zeros ($A_3 = \mathbf{0}$). Block (A_4) is a matrix of size ($m \times K_B$). The elements of (A_4) are ($A_4 = \sum_{k'}^{K_B} \sum_j^m P^{j,k'} \times (\mathsf{d})_j$). The block matrix ($\mathbf{A}$) is of size ($(n+m) \times (K_A + K_B)$).

The tensor operator ($\mathbf{\Delta}$) is given as ($\mathbf{\Delta} = \sum_k^{K_A} \sum_i^n C^{i,k} p(s_A^i)(\mathsf{d})_i \oplus$)
($\sum_{k'}^{K_B} \sum_j^m C^{j,k'} p(s_B^j(\mathsf{d})_j$), where ($C_{K_A}^k = \frac{K_A!}{k!(K_A-k)!}$) , discarding the order of the subsets,i.e. (i) is not ordered, is a combination of possible causal conceptual subsets ($s_A^i \subset L_A$) that can be added to the basic syntax in order to get the perceived reasoning. ($p(s_A^i) = \frac{C_{K_A}^k}{K_A}$) is the probability that such a transformation is adopted. Similarly, ($C_{K_B}^{k'} = \frac{K_B!}{k'!(K_B-k')!}$), discarding the order of the subsets, i.e. (j) is not ordered, is a combination of possible causal conceptual subsets ($s_B^j \subset L_B$) that can be added to the basic syntax in order to get the perceived reasoning. ($p(s_B^j) = \frac{C_{K_B}^{k'}}{K_B}$) is the probability that such a transformation is adopted. The operator tensor ($\mathbf{\Delta}$) is of

size $((n+m) \times (K_A + K_B))$ consists of (4) blocks $(\Delta_1, \Delta_2, \Delta_3, \Delta_4)$ of matrices similar to matrix $(\mathbf{A})$. Block (Δ_1) is a matrix of size $(n \times K_A)$. The elements of (Δ_1) are $(\Delta_1 = \sum_k^{K_A} \sum_i^n \sum_k^{K_A} \sum_i^n C^{i,k} p(s_A^i))$. Block (Δ_2) is a matrix of size $(n \times K_B)$ with all elements zeros $(\Delta_2 = \mathbf{0})$. Block (Δ_3) is a matrix of size $(m \times K_A)$ with all elements zeros $(\Delta_3 = \mathbf{0})$. Block (Δ_4) is a matrix of size $(m \times K_B)$. The elements of (Δ_4) are $(\Delta_4 = \sum_{k'}^{K_B} \sum_j^m C^{j,k'} p(s_B^j))$. The matrix $(\mathbf{\Delta})$ is of size $((n+m) \times (K_A + K_B))$.

Causal conceptual Syntax manifolds are hyper-planes. They are constructed by causal conceptual tensor fields. The following demonstrates the mathematical formulation leading to the built up of causal conceptual tensor fields. A syntax tensor is formulated as $(\xi = \bar{\xi} \otimes \mathbf{\Delta})$. The segment $(\bar{\xi})$ is formulated as $(\bar{\xi} = ((\mathbf{A_1 A_2}) \mid ((\mathbf{A_3 A_4})))$. The operator, $(\mathbf{\Delta})$ is formulated as $(\mathbf{\Delta} = ((\mathbf{\Delta_1 \Delta_2}) \mid ((\mathbf{\Delta_3 \Delta_4})))$. A conceptual tensor field can be obtained by transforming the segment $(\bar{\xi})$ and transforming the segment $(\mathbf{\Delta})$, and it is formulated as $(\mathbf{T}(\bar{\xi}, \mathbf{\Delta}) = \mathbf{T}(\bar{\xi}) \otimes \mathbf{T}(\mathbf{\Delta}))$, where $(\mathbf{T}(\bar{\xi}, \mathbf{\Delta}))$ represents a conceptual tensor field, $(\mathbf{T}(\bar{\xi}))$ is a transformation of $(\bar{\xi})$, and $(\mathbf{T}(\mathbf{\Delta})$ is the transformation of the operator $(\mathbf{\Delta})$.

$(\mathbf{T}(\bar{\xi}))$ is given as $(\mathbf{T}(\bar{\xi}) = ((\bar{\mathbf{A_1}}\mathbf{A_2})) \mid ((\mathbf{A_3}\bar{\mathbf{A_4}})))$, where $(\bar{\mathbf{A_1}})$ is a tensor of size $(n \times K_A)$ with elements $(\bar{\mathbf{A_1}} = \bar{a}_{1i,k} = P_{K_A}^i \bar{\mathsf{d}})_i; i = 1, \cdots, n; k = 1, \cdots, K_A)$, where $(\bar{\mathbf{A_1}} \neq \mathbf{A_1})$, and $((\bar{\mathsf{d}})_i \neq (\mathsf{d})_i)$. $(\mathbf{A_2})$ is a tensor of size $(n \times K_B)$, with elements $(\mathbf{A_2} = \mathbf{0})$. $(\mathbf{A_3})$ is a tensor of size $(m \times K_A)$, with elements $(\mathbf{A_3} = \mathbf{0})$. $(\bar{\mathbf{A_4}})$ is a tensor of size $(m \times K_B)$ with elements $(\bar{\mathbf{A_4}} = \bar{a}_{4j,k'} = P_{K_B}^j \bar{\mathsf{d}})_j; j = 1, \cdots, m; k' = 1, \cdots, K_B)$, where $(\bar{\mathbf{A_4}} \neq \mathbf{A_4})$, and $((\bar{\mathsf{d}})_j \neq (\mathsf{d})_j)$.

Similarly, $(\mathbf{T}(\mathbf{\Delta})$ is obtained as $(\mathbf{T}(\mathbf{\Delta} = ((\bar{\mathbf{\Delta_1}}\mathbf{\Delta_2})) \mid ((\mathbf{\Delta_3}\bar{\mathbf{\Delta_4}})))$, where $(\bar{\mathbf{\Delta_1}})$ is a tensor of size $(n \times K_A)$ with elements $(\bar{\mathbf{\Delta_1}} = \bar{\delta}_{1i,k} = C_{K_A}^i p_{s_A^i} \bar{\mathsf{d}})_i; i = 1, \cdots, n; k = 1, \cdots, K_A)$, where $(\bar{\mathbf{\Delta_1}} \neq \mathbf{\Delta_1})$, and $((\bar{\mathsf{d}})_i \neq (\mathsf{d})_i)$. $(\mathbf{\Delta_2})$ is a tensor of size $(n \times K_B)$, with elements $(\mathbf{\Delta_2} = \mathbf{0})$. $(\mathbf{\Delta_3})$ is a tensor of size $(m \times K_A)$, with elements $(\mathbf{\Delta_3} = \mathbf{0})$. $(\bar{\mathbf{\Delta_4}})$ is a tensor of size $(m \times K_B)$ with elements $(\bar{\mathbf{\Delta_4}} = \bar{\delta}_{4j,k'} = C_{K_B}^j p_{s_B^j} \bar{\mathsf{d}})_j; j = 1, \cdots, m; k' = 1, \cdots, K_B)$, where $(\bar{\mathbf{\Delta_4}} \neq \mathbf{\Delta_4})$, and $((\bar{\mathsf{d}})_j \neq (\mathsf{d})_j)$. The conceptual tensor field, $(\mathbf{T}(\bar{\xi}, \mathbf{\Delta}))$ is formulated as $(\mathbf{T}(\bar{\xi}, \mathbf{\Delta}) = \mathbf{T}(\bar{\xi}) \otimes \mathbf{T}'(\mathbf{\Delta}))$, where $(\mathbf{T}'(\mathbf{\Delta}))$ is the transpose of matrix $(\mathbf{T}(\mathbf{\Delta}))$. $(\mathbf{T}(\bar{\xi}, \mathbf{\Delta}))$ is tensor of size $([(n+m) \times (K_A + K_B)] \times [(K_A + K_B) \times (n+m)] = [(n+m) \times (n+m)])$.

3 Causal Action

Action in the context of this paper refers to all physical movements of a person (arms,hands, legs, feet, and body movements), that for a goal are either done for the purpose of consumption, or production. Figure below shows different facets of action. In this chapter, it is shown how physical movements are translated into action. once the relationship between physical movements, and action is established, then it is demonstrated how syntax, (ξ) is related to action, using the causal cuboid as a tool for this purpose.

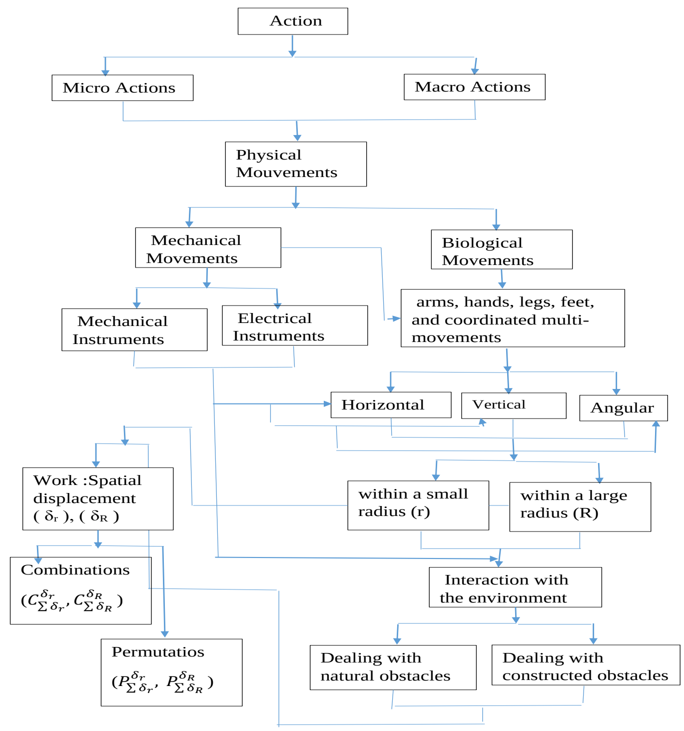


Figure 13. Action diagram

As is shown in Figure 13, action is divided into two categories, micro actions, and macro actions Micro actions are physical movements that are done within a circle of small radius, denoted as (γ_r). Macro actions are physical movements that are done within a circle of large radius denoted as (γ_R). Physical movements are differentiated into mechanical movements and biological movements. Mechanical movements are movements that are produced by manipulating mechanical or electrical instruments. The particularity of these types of movements is that these movements are performed within small or large zones. Interaction with machines involves (horizontal, vertical, and angular) movements that produce short range or long range actions. Biological movements (arms, hands, legs, feet) are specified by the form of the movement, (horizontal, vertical, and angular), since these motions forms can be stratified as being performed within a small and or large radius they are considered as work. Both interaction with machines, and biological movements are interactions with the environment. This is identified as either the ability to deal with natural obstacles or the ability to deal with constructed obstacles. In all cases, movement constituting action is a spatial displacement $(\Gamma(\gamma_r, \gamma_R))$, either within a small radius (r) or within a large radius (R).

There are combinations, and permutations of (γ_r), and (γ_R), $(C^{\gamma_r}_{\sum\gamma_r}, C^{\gamma_R}_{\sum\gamma_R}; (r, R) \in \Re)$ representing the twelve different types of causal actions $(\Gamma(\gamma_r, \gamma_R))$, shown in Figure 2. The (12) types of actions constitute the (12) axes of the causal cuboid discussed in the previous section.

Thus perceived, action is considered as a force represented in general by a vector in an identified coordinate system. When syntax constitutes the bases of action, then the force produced by action should be represented as a tensor due to the moving non-vertical coordinates representing the causal cuboid axes caused by different perceptions of a syntax, (after transformation). Thus each axis of the causal cuboid representing action is a tensor representation. One important significance is that both the demand axis given as action, $(\Gamma_q(\gamma_r, \gamma_R))$, where (q) stands for demand or consumption, and the price axis given as action, $(\Gamma_p(\gamma_p, \gamma_R))$, where (p) stands for price, are tensors. The action tensor can be measured as a metric tensor. Thus all axes of the causal cuboid are metric tensors.

Causal conceptual hyperplanes are related to actions ending in consumption, and price setting, $(\Gamma_q(\gamma_r, \gamma_R))$, and $(\Gamma_p(\gamma_p, \gamma_R))$, described earlier. To make this connection, a simplification is applied. Action is considered to constitute circular zones of small and large radius, (r,R) denoted as (γ_r, γ_R), where (γ_r) is work performed in a circular zone of radius (r), and (γ_R) is work performed in a circular zone of radius (R). A homeomorphism from a syntax to work is as given in the diagram in Figure 2, it is possible to find a transformation from syntax to action within circular zones,, $((\xi, \Delta) \rightarrow (\gamma_r, \gamma_R))$, is denoted as a transformation function (f_γ). Function (f_γ) is defined for the ten different causal actions related the axis (q) demand, and the axis (p), price are functions of the other ten axes. Consequently, there is a relationship between causal conceptual syntax tensor, $(\mathbf{T}(\bar{\xi}, \mathbf{\Delta}) \rightarrow \Gamma(\gamma_r, \gamma_R))$, where (Γ) and the causal action tensor. Therefore, the ten remaining action tensors are described as functions of actions that lead to consumption, (q), and production $(\mathfrak{P})$, as production relates to price setting alongside of consumption. The ten transformation functions are formulated as follows:

Axis (1) is the consumption axis (q). The homeomorphism $(\Gamma(\xi, \Delta) \rightarrow \Gamma(\gamma_r, \gamma_R)_q)$ is done through the homeomorphic function, $(f_1(\gamma))$ such that $(f_1(\gamma) = f_q(\Gamma(\gamma_r, \gamma_R))_q + \omega_{\mathfrak{P}} f_{\mathfrak{P}}(\Gamma(\gamma_r, \gamma_R))_{\mathfrak{P}})$ where $(f_q(\Gamma(\gamma_r, \gamma_R))_q)$ corresponds to any of the tens causal cuboid axes that relate to (q), and $(f_{\mathfrak{P}}(\Gamma(\gamma_r, \gamma_R))_{\mathfrak{P}})$ corresponds to any of the ten axes relating to production that impacts the price setting procedure. $(\omega_{\mathfrak{P}})$ is the weight given to the production function,$(f_{\mathfrak{P}})$ that represents the impact of this function on the quantity demanded.

Axis (2) is the price axis (p). The homeomorphism $(\Gamma(\xi, \Delta) \rightarrow \Gamma(\gamma_r, \gamma_R)_{\mathfrak{P}})$ is done through the homeomorphic function, $(f_2(\gamma))$ such that $(f_2(\gamma) = f_{\mathfrak{P}}(\Gamma(\gamma_r, \gamma_R))_{\mathfrak{P}} + \omega_q f_q(\Gamma(\gamma_r, \gamma_R))_q)$ where $(f_{\mathfrak{P}}(\Gamma(\gamma_r, \gamma_R))_{\mathfrak{P}})$ corresponds to any of the tens causal cuboid axes that relate to $(\mathfrak{P})$, production activities, and $(f_q(\Gamma(\gamma_r, \gamma_R))_q)$ corresponds to any of the ten axes relating to quantity consumed that can affect the price setting process. (ω_q) is the weight given to the quantity of consumption function that represents the impact of this function on the price determination.

Axis (3) is a homeomorphism from syntax to work, $(\Gamma(\xi, \Delta)_q \rightarrow \Gamma(\gamma_r, \gamma_R)_q)$, where $(\Gamma(\xi, \Delta)_q)$ is the action identified in the syntax $((\xi, \Delta)_q)$ that is related to the quantity of consumption determination $(\Gamma(\gamma_r, \gamma_R)_q)$. $(\Gamma(\gamma_r, \gamma_R))$ is action that is a function of work $((\gamma_r, \gamma_R))$. Both (γ_r), and (γ_R) are derived from syntax, $((\xi, \Delta))$ $(f_3(\gamma))$ is a causal action function of actions of type relating to the first axis, (q) of the causal cuboid, and each $(\{\})$ represent sets of circular zones of actions. The work function is given as $((f_3(\gamma)(\gamma_r, \gamma_R))_q = [(\{\gamma_r\} \cup \{\gamma_R\}) \oplus (\{\gamma_r\} \cap \{\gamma_R\})]_q)$. $(\{\gamma_r\} \cup \{\gamma_R\}))$ is the union of sets of small zonr , and large zone work. $((\{\gamma_r\} \cap \{\gamma_R\})]_q)$ is the set of work that is common to both (r), and (R) zones. Let $([(\{\gamma_r\} \cup \{\gamma_R\}) \oplus (\{\gamma_r\} \cap \{\gamma_R\})]_q)$ be designated as (A_q), and summarize as $(f_3(\gamma) = A_q)$.

Axis (4), is a homeomorphism from syntax to work, $(\Gamma(\xi, \Delta)_{\mathfrak{P}} \to \Gamma(\gamma_r, \gamma_R)_{\mathfrak{P}})$, where $(\Gamma(\xi, \Delta)_{\mathfrak{P}})$ is the action identified in the syntax $((\xi, \Delta)_{\mathfrak{P}})$ that is related to production activities that determine price setting process, $(\Gamma(\gamma_r, \gamma_R)_{\mathfrak{P}})$. $(\Gamma(\gamma_r, \gamma_R)_{\mathfrak{P}})$ is action that is a function of work $((\gamma_r, \gamma_R))$ related to production activities. Both (γ_r), and (γ_R) are derived from syntax, $((\xi, \Delta))$. $(f_4(\gamma))$ is a causal action function of action of type relating to the second axis of the causal cuboid, the price axis, and each $(\{\})$ represents sets of circular zones of actions. The work function is formulated as $((f_{\gamma_4}(\gamma_r, \gamma_R))_{\mathfrak{P}} = [(\{\gamma_r\} \cup \{\gamma_R\}) \oplus (\{\gamma_r\} \cap \{\gamma_R\})]_{\mathfrak{P}})$. The subscript $(\mathfrak{P})$, represents production. Let $([(\{\gamma_r\} \cup \{\gamma_R\}) \oplus (\{\gamma_r\} \cap \{\gamma_R\})]_{\mathfrak{P}})$ be designated as $(B_{\mathfrak{P}})$.

Axis (5), is a homeomorphism from syntax to work, $(\Gamma(\xi, \Delta)_{\mathfrak{P}} \to \Gamma(\gamma_r, \gamma_R)_{q,\mathfrak{P}})$, where $(\Gamma(\xi, \Delta)_{q,\mathfrak{P}})$ is the action identified in the syntax $((\xi, \Delta)_{q,\mathfrak{P}})$ that is related to the quantity of consumption, and production activities that determine the quantity demanded, and the price setting, $(\Gamma(\gamma_r, \gamma_R)_{q,\mathfrak{P}})$. $(\Gamma(\gamma_r, \gamma_R)_{q,\mathfrak{P}})$ is action that is a function of work $((\gamma_r, \gamma_R))$ related to consumption, and production activities. The action function is formulated as, $((f_{\gamma_5}(\gamma_r, \gamma_R))_{q,\mathfrak{P}} = A_q \oplus B_{\mathfrak{P}} = [(\{\gamma_r\} \cup \{\gamma_R\}) \oplus (\{\gamma_r\} \cap \{\gamma_R\})]_q \oplus [(\{\gamma_r\} \cup \{\gamma_R\}) \oplus (\{\gamma_r\} \cap \{\gamma_R\})]_{\mathfrak{P}})$. The summarized formulation is $(f_{\gamma_5} = A_q \oplus B_{\mathfrak{P}})$.

Axis (6), is a homeomorphism from syntax to work, $(\Gamma(\xi, \Delta)_{\mathfrak{P}} \to \Gamma(\gamma_r, \gamma_R)_{q,\mathfrak{P}})$, where $(\Gamma(\xi, \Delta)_{q,\mathfrak{P}})$ is the action identified in the syntax $((\xi, \Delta)_{q,\mathfrak{P}})$ that is related to quantity of consumption, and production activities in the way that the consumption activities are embedded in the production activities. This means that individual possesses a knowledge of production activities in order to determine the action types, that determine the quantity demanded, $(\Gamma(\gamma_r, \gamma_R)_q)$. $(\Gamma(\gamma_r, \gamma_R)_{q,\mathfrak{P}})$ is an embedded action that is a function of work towards consumption that incorporates production activities $((\gamma_r, \gamma_R)_{q,\mathfrak{P}})$, where work towards consumption is a function of production work. The action function is formulated as, $((f_{\gamma_6}(\gamma_r, \gamma_R))_{q,\mathfrak{P}} = A_q \circ B_{\mathfrak{P}} = B_{\mathfrak{P}}(A_q) = [(\{\gamma_r\})_q \cap (\{\gamma_r\})_{\mathfrak{P}} \cup (\{\gamma_R\})_q \cap (\{\gamma_R\})_{\mathfrak{P}}]_q \oplus [(\{\gamma_r\})_q \cap (\{\gamma_r\})_p \cap (\{\gamma_R\})_q \cap (\{\gamma_R\})_{\mathfrak{P}}]_{\mathfrak{P}})$. The summarized formulation is $(f_{\gamma_6} = A_q \circ B_{\mathfrak{P}})$.

Axis (7), is a homeomorphism from syntax to work, $(\Gamma(\xi, \Delta)_q \to \Gamma(\gamma_r, \gamma_R)_q)$, where $(\Gamma(\xi, \Delta)_q)$ is a subset of the set of actions identified in the syntax $((\xi, \Delta)_q)$ that is related to quantity of consumption. The action function is formulated as, $((f_{\gamma_7}(\gamma_r, \gamma_R))_q = a \centerdot A_q)$, where (a) is a fraction of action used for the purpose of consumption. The summarized formulation is $(f_{\gamma_7} = a \centerdot A_q)$.

Axis (8), is a homeomorphism from syntax to work, $(\Gamma(\xi,\Delta)_q \to \Gamma(\gamma_r,\gamma_R)_q)$, where $(\Gamma(\xi,\Delta)_q)$ is a probabilistic occurrence of the set of actions identified in the syntax $((\xi,\Delta)_q)$ that is related to quantity of consumption. The action function is formulated as, $((f_{\gamma_8}(\gamma_r,\gamma_R))_q = \alpha \centerdot A_q)$, where (α) is the probability of action occurring with the purpose of consumption. The summarized formulation is, $(f_{\gamma_8} = \alpha \centerdot A_q)$.

Axis (9), is a homeomorphism from syntax to work, $(\Gamma(\xi,\Delta)_{\mathfrak{P}} \to \Gamma(\gamma_r,\gamma_R)_{\mathfrak{P}})$, where $(\Gamma(\xi,\Delta)_{\mathfrak{P}})$ is a subset of the set of actions identified in the syntax $((\xi,\Delta)_{\mathfrak{P}})$ that is related to production activities that is the basis of the price setting. The action function is formulated as, $((f_{\gamma_9}(\gamma_r,\gamma_R))_{\mathfrak{P}} = b \centerdot B_{\mathfrak{P}})$, where $(\mathfrak{P})$ is a fraction of action used for the purpose of production. The summarized formulation is, $(f_{\gamma_9} = b \centerdot B_{\mathfrak{P}})$.

Axis (10), is a homeomorphism from syntax to work, $(\Gamma(\xi,\Delta)_{\mathfrak{P}} \to \Gamma(\gamma_r,\gamma_R)_{\mathfrak{P}})$, where $(\Gamma(\xi,\Delta)_{\mathfrak{P}})$ is a subset of the set of actions identified in the syntax $((\xi,\Delta)_{\mathfrak{P}})$ that is related to production activities that is the basis of price setting. The action function is formulated as, $((f_{\gamma_{10}}(\gamma_r,\gamma_R))_{\mathfrak{P}} = \beta \centerdot B_{\mathfrak{P}})$, where (β) is the probability of the set of actions occurring for the purpose of production. The summarized formulation is, $(f_{\gamma_{10}} = \beta \centerdot B_{\mathfrak{P}})$.

Axis (11), is a homeomorphism from syntax to work, $(\Gamma(\xi,\Delta)_{q,\mathfrak{P}} \to \Gamma(\gamma_r,\gamma_R)_{q,\mathfrak{P}})$, where $(\Gamma(\xi,\Delta)_{q,\mathfrak{P}})$ includes a subset of the set of actions identified in the syntax $((\xi,\Delta)_{q,\mathfrak{P}})$ that are related to the quantity of consumption, and subsets of actions that are related to production activities. The action function is formulated as, $((f_{\gamma_{11}}(\gamma_r,\gamma_R))_{q,\mathfrak{P}} = a \centerdot A_q \oplus b \centerdot B_{\mathfrak{P}})$. The coefficients (a,b) indicate the number of subsets of each type of action, either for the purpose of consumption or for the purpose of production. The summarized formulation is, $(f_{\gamma_{11}} = a \centerdot A_q \oplus b \centerdot B_{\mathfrak{P}})$.

Axis (12), is a homeomorphism from syntax to work, $(\Gamma(\xi,\Delta)_{q,\mathfrak{P}} \to \Gamma(\gamma_r,\gamma_R)_{q,\mathfrak{P}})$, where $(\Gamma(\xi,\Delta)_{q,\mathfrak{P}})$ includes the probability of the set of actions relating to the quantity of consumption,(α) and the probability of the set of actions relating to production, (β) identified in the syntax $((\xi,\Delta)_{q,\mathfrak{P}})$. The action function is formulated as, $((f_{\gamma_{12}}(\gamma_r,\gamma_R))_{q,\mathfrak{P}} = \alpha \centerdot A_q \oplus \beta \centerdot B_{\mathfrak{P}})$. The summarized formulation is, $(f_{\gamma_{12}} = \alpha \centerdot A_q \oplus \beta \centerdot B_{\mathfrak{P}})$.

Let $(\mathbb{H}(\xi,\Delta))$ represent the causal conceptual tensor hyperplane constructed by syntax tensors. (ξ) represent the syntax tensor, and (Δ) are the operator tensors that represent the dialectic or the evolutionary possible modifications to the base syntax tensor (ξ). The homeomorphism is represented as, $(\mathbb{H}(\xi,\Delta) \xrightarrow{S^3(I,v,F)} \Gamma(\gamma_r,\gamma_R))$. $(\Gamma(\gamma_r,\gamma_R))$ is the action tensor identified by the syntax tensor hyperplane $(\mathbb{H}(\xi,\Delta))$. (γ_r) is the work represented by a circular zone with a small radius (r). (γ_R) is the work represented by a circular zone with a large radius (R). The function used for this homeomorphism is a (3) dimensional spheroid denoted as (S^3). This cohomology is done using 3-D spheres with bases defined as (I,v,F), denoted as $(S^3(I,v,F))$. The bases (I,v,F) are defined as (I) that represents intensity. Intensity is defined as the intensity of action, specified by the syntax. Intensity in this context is the number of adjectives/adverbs in the syntax. The second basis is (v) representing velocity. In this context velocity is the number of predicates or actions verbs per syntax used. The third basis is (F), frequency. Frequency in this context is the number of times an action verb is repeated in a syntax. Figure 14, demonstrates the cohomology diagram, between a causal conceptual syntax hyperplane , ($\mathbb{H}(\xi, \Delta)$), and the action tensor, ($\Gamma(\gamma_r, \gamma_R)$), through spheroids, denoted by ($S^3(I, v, F)$) given as ($\mathbb{H}(\xi, \Delta) \xrightarrow{S^3(I,v,F)} \Gamma(\gamma_r, \gamma_R)$).

Action is denoted as consisting of work within circular zones of small radius (r), (γ_r), and work within circular zones of large radius (R), (γ_R). Work done in a small radius zone is refined by identifying work that is extracted from the two groups (s^i_A), and (s^j_B) as ($\gamma_r = \{\gamma_{r_1}, \cdots, \gamma_{r_n}\} \cup \{\gamma_{r'_1}, \cdots, \gamma_{r'_m}\}; n \neq m$), and where ($\gamma_{r_i} \in s^i_A; s^i_A \subset L_A$), and ($\gamma_{r'_j} \in s^j_B;; s^j_B \subset L_B$), (r'). Work done in a large radius zone is refined by identifying work that is extracted from the two groups (s^i_A), and (s^j_B) is denoted as ($\gamma_R = \{\gamma_{R'_1}, \cdots, \gamma_{R'_n}\} \cup \{\gamma_{R_1}, \cdots, \gamma_{R_m}\}; n \neq m$), and where ($\gamma_{R_i} \in s^i_A; s^i_A \subset L_A$), and ($\gamma_{R'_j} \in s^j_B;; s^j_B \subset L_B$). The above categorization of work can be incorporated in the formulation of action of each axis of the causal cuboid.

In order to implement the homeomorphic mapping, from a syntax hyperplane to an spheroid($\mathbb{H}(\xi, \Delta) \xrightarrow{S^3(I,v,F)} \Gamma(\gamma_r, \gamma_R)$), the following modifications are required. It is assumed that each spheroid can be identified within specific zones of action. Action is categorized into specific categories. The action categories are given in Figure 14. As is demonstrated in Figure 14, there are (14) types of specific actions that are identified. These types of actions are: category 1) ($\Gamma(\gamma_r, \gamma_{r'})$) is action that consists of short range work, (small movements) within a small zone of radius (r), ($r \in s^i_A$),($s^i_A \subset L_A$), and short range work within a large zone of radius (R), ($r' \in s^j_B$),($s^j_B \subset L_B$). Category 2) ($\Gamma(\gamma_R, \gamma_{R'})$), is action that consists of long range work, (large movements) within a small zone of radius (r), and long range work within a large zone of radius (R),($R \in s^j_B$),($s^j_B \subset L_B$) ; category 3) ($\Gamma(\gamma_r, \gamma_R)$) is action that consists of short range work, (small movements) within a small zone of radius (r), and long range work within a large zone of radius (R); category 4) ($\Gamma(\gamma_r, \gamma_{R'})$) is action that consists of short range work, (small movements) within a small zone of radius (r), and long range work within a small zone of radius (r),($R' \in s^i_A$),($s^i_A \subset L_A$); category 5) ($\Gamma(\gamma_{r'}, \gamma_R)$) is action that consists of short range work, (small movements) within a large zone of radius (R), and long range work within a large zone of radius (R); category 6) ($\Gamma(\gamma_{r'}, \gamma_{R'})$) is action that consists of short range work, (small movements) within a large zone of radius (R), and long range work within a small zone of radius (r).

Category 7) ($\Gamma(\gamma_r, \gamma_R) \cap \Gamma(\gamma_r, \gamma_{R'})$) is action that consists of the intersection of category (3), and category (4). These are actions that relate to short range work, (small movements) within a small zone of radius (r); 8) category ($\Gamma(\gamma_{r'}, \gamma_R) \cap \Gamma(\gamma_{r'}, \gamma_{R'})$) is action that consists of the intersection of category (5), and category (6). These are actions that relate to short range work, (small movements) within a large zone of radius (R); category 9) ($\Gamma(\gamma_r, \gamma_R) \cup \Gamma(\gamma_r, \gamma_{R'})$) is action that consists of the union of category (3), and category (4). These are actions that relate to all short range work, (small movements) within a small zone of radius (r), and actions that relate all long range work, (large movements) within a large sone of radius (R); category 10) ($\Gamma(\gamma_{r'}, \gamma_R) \cup \Gamma(\gamma_{r'}, \gamma_{R'})$) is action that consists of the union of category (5), and category (6). These are actions that relate to short range work, (small movements) within a large zone of radius (R), and actions that relate all long range work, (large movements) within a large zone of radius (R).

category 11) $(\Gamma(\gamma_r, \gamma_R) \cap \Gamma(\gamma_R, \gamma_{R'}))$ is action that consists of the intersection of category (3), and category (2). These are actions that relate to long range work, (large movements) within a large zone of radius (R), and actions that relate all long range work, (large movements) within a large zone of radius (R); category 12) $(\Gamma(\gamma_r, \gamma_R) \cap \frac{\Gamma(\gamma_r, \gamma_{r'})}{\Gamma(\gamma_r, \gamma_{r'})})$ is action that consists of the intersection of category (3), and the ratio of category (4) over category (1). These are actions that relate to short range work, (small movements) within a small zone of radius (r), and actions that relate all long range work, (large movements) within a large zone of radius (R), with each element of this action type multiplied by the fraction $(\frac{4}{1})$. Category 13) $(\Gamma(\gamma_r, \gamma_R) \cap \Gamma(\gamma_r, \gamma_{r'}))$ is action that consists of the intersection of category (3), and category (1). These are actions that relate to short range work, (small movements) within a small zone of radius (r); category 14) $(\Gamma(\gamma_r, \gamma_R) \cup \frac{\Gamma(\gamma_R, \gamma_{R'})}{\Gamma(\gamma_r, \gamma_{r'})})$ is action that consists of the union of category (3), and the ratio of category (2) over category (1). These are actions that relate to short range work, (small movements) within a small zone of radius (r), and actions that relate all long range work, (large movements) within a large zone of radius (R), with each element of this action type augmented by the fraction $(\frac{2}{1})$. An spheroid can be found for each of the (14) action types introduced. Figure 14 sums up the homeomorphic mapping in a graphical format.

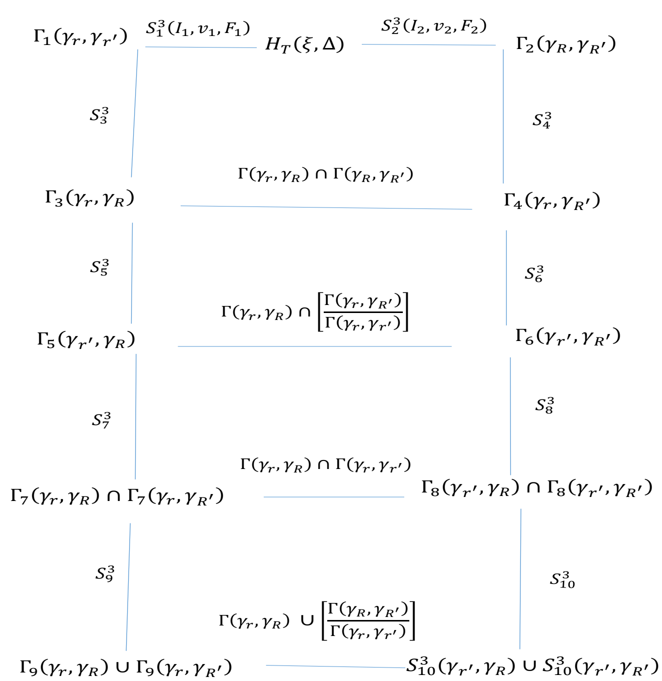


Figure 14. Cohomology of causal conceptual hyperplanes

An example of the homeomorphic mapping between a sample of action types $(\Gamma(\gamma., \gamma.))$, as they are introduced in Figure 14, and spheroids as spherical functions (S^3), is given in Figures 15, and 16. Figures 15, and Figure 16, demonstrate variations in shape of the action spheres in 3D. Depending on the nature of the action, spheres are either stretched or compacted.

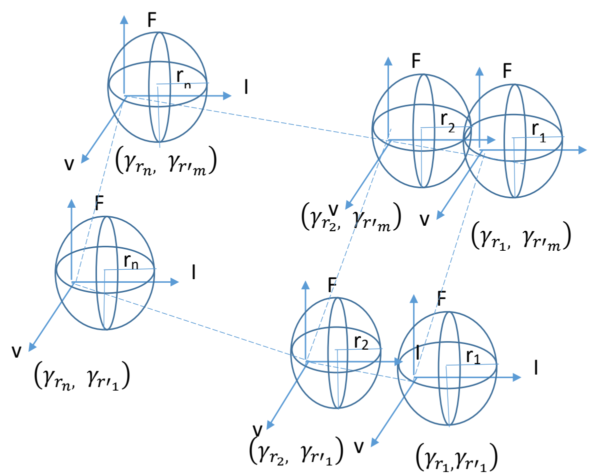


Figure 15. Spheres of small radii (r)

and

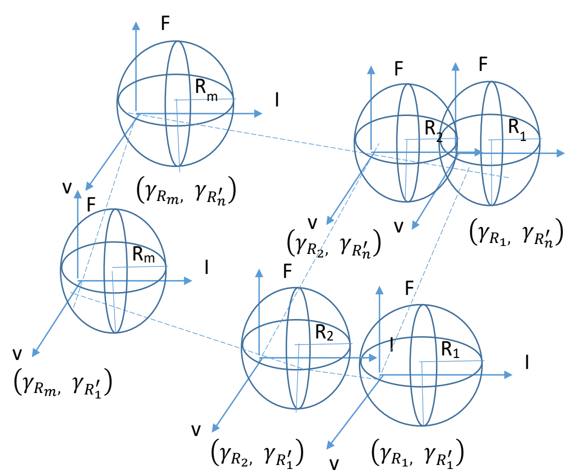


Figure 16. Spheres of large radii (R)

The graphical representation of transition from a tensor based causal conceptual hyperplane to a 3D spherical space to a causal cuboid denoted as (V_c) as is shown in Figure 17. The spheroid, (S^3) is formulated with respect to radius (r), and (R) as follows: ($S_r^3 = \mathbf{r^2} = \mathbf{a_r^2 I_r^2} \oplus \mathbf{b_r^2 v_r^2} \oplus \mathbf{c_r^2 F_r^2}$), and ($S_R^3 = \mathbf{R^2} = \mathbf{(aa)_R^2 I_R^2} \oplus \mathbf{(bb)_R^2 v_R^2} \oplus \mathbf{(cc)_R^2 F_R^2}$), where ($\mathbf{a_r}, \mathbf{b_r}, \mathbf{c_r}$) are the parameters corresponding to spheroids of radius (r), and ($\mathbf{aa_R}, \mathbf{bb_R}, \mathbf{cc_R}$) are the parameters corresponding to spheroids of radius (R).

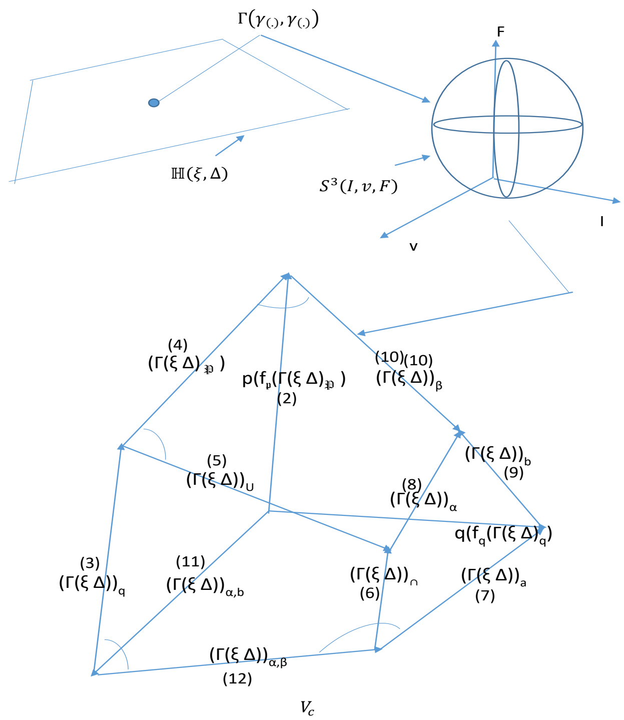


Figure 17. Transition from a causal conceptual hyperplane to an spheroid to a causal cuboid

The action tensor ($\Gamma(\textbf{.})$) is defined as an $((n+m)\times(n+m))$ dimensional tensor, ($\mathbf{\Gamma}$), written as a tensor divided into 4 blocks, ($\mathbf{\Gamma} = [\mathbf{\Gamma}_1, \mathbf{\Gamma}_2, \mathbf{\Gamma}_3, \mathbf{\Gamma}_4]$). The first block, ($\mathbf{\Gamma}_1 = \mathbf{tr}(\mathbf{r})$) is an $(n \times n)$ tensor, where the trace of (r), ($\mathbf{tr}(\mathbf{r})$) is defined as $\mathbf{tr}(\mathbf{r}) = \begin{cases} \sum_i \sum_j (a_{i,j}^2 + b_{i,j}^2 + c_{i,j}^2) & \forall i = j \\ 0 & i \neq j \end{cases}$, and (a, b, c) are the coefficients of the spheroid that represents work within a small radius (r), included in the syntax segment ($s_A^i \in L_A$).. The second block, ($\mathbf{\Gamma}_2 = \mathbf{tr}(\mathbf{r'})$) is an $(n \times m)$ tensor, where the trace of (r'), ($\mathbf{tr}(\mathbf{r'})$) is defined as $\mathbf{tr}(\mathbf{r'}) = \begin{cases} \sum_i \sum_j (a_{i,j}'^2 + b_{i,j}'^2 + c_{i,j}'^2) & \forall i = j \\ 0 & i \neq j \end{cases}$, and (a', b', c') are the coefficients of the spheroid that represents work within a small radius (r'), included in the syntax segment ($s_B^j \in L_B$). The third block, ($\mathbf{\Gamma}_3 = \mathbf{tr}(\mathbf{R'})$)

is an $(m \times n)$ tensor, where the trace of (R'), $(\mathbf{tr}(\mathbf{R}'))$ is defined as $\mathbf{tr}(\mathbf{R}') = \begin{cases} \sum_i \sum_j (aa'^2_{i,j} + bb'^2_{i,j} + cc'^2_{i,j}) & \forall i = j \\ 0 & i \neq j \end{cases}$, and (aa', bb', cc') are the coefficients of the spheroid that represents work within a large radius (R'), included in the syntax segment $(s^i_A \in L_A)$. The fourth block, $(\mathbf{\Gamma}_4 = \mathbf{tr}(\mathbf{R}))$ is an $(m \times m)$ tensor, where the trace of (R), $(\mathbf{tr}(\mathbf{R}))$ is defined as $\mathbf{tr}(\mathbf{R}) = \begin{cases} \sum_i \sum_j (aa^2_{i,j} + bb^2_{i,j} + cc^2_{i,j}) & \forall i = j \\ 0 & i \neq j \end{cases}$, and (aa, bb, cc) are the coefficients of the spheroid that represents work within a large radius (R), included in the syntax segment $(s^j_B \in L_B)$. The action tensors $(\Gamma(\centerdot))$ is given below:

$$\Gamma(\centerdot) = \begin{array}{c|ccc|ccc|} & & & \Gamma_1 & & & \Gamma_2 \\ & 1 & \cdots & n & 1 & \cdots & m \\ 1 & tr(r)_{1,1} & \cdots & 0 & tr(r')_{1,1} & \cdots & 0 \\ \vdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ n & 0 & \cdots & t(r)_{n,n} & 0 & \cdots & t(r')_{n,m} \\ \hline 1 & tr(R')_{1,1} & \cdots & 0 & tr(R)_{1,1} & \cdots & 0 \\ \vdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ m & 0 & \cdots & tr(R')_{m,n} & 0 & \cdots & tr(R)_{m,m} \\ & & & \Gamma_3 & & & \Gamma_4 \end{array}$$

The homeomorphic actions given in Figure 14, can be defined based on the action tensor blocks as follows: 1) $(\mathbf{\Gamma}(\gamma_\mathbf{r}, \gamma_{\mathbf{r}'}))$ is an action tensor that combines work within a small radius $(r \subset s^i_A)$, and work within a small radius $(r' \subset s^j_B)$, denoted as $(\mathbf{\Gamma}(\gamma_\mathbf{r}, \gamma_{\mathbf{r}'}) = \mathbf{tr}(\mathbf{r}), \mathbf{tr}(\mathbf{r}') = (\mathbf{\Gamma_1}, \mathbf{\Gamma_2}); (\mathbf{\Gamma_3} = \mathbf{\Gamma_4} = \mathbf{0}))$ is a tensor of dimension $((n \times m) = (n \times n) \times (n \times m))$. The tensor is shown below:

$$\Gamma_1 = \begin{array}{c|ccc|ccc|} & & & \Gamma_1 & & & \Gamma_2 \\ & 1 & \cdots & n & 1 & \cdots & m \\ 1 & tr(r)_{1,1} & \cdots & 0 & tr(r')_{1,1} & \cdots & 0 \\ \vdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ n & 0 & \cdots & t(r)_{n,n} & 0 & \cdots & t(r')_{n,m} \\ \hline 1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ m & 0 & \cdots & 0 & 0 & \cdots & 0 \\ & & & \Gamma_3 & & & \Gamma_4 \end{array}$$

2) $(\mathbf{\Gamma}(\gamma_\mathbf{R}, \gamma_{\mathbf{R}'}))$ is an action tensor that combines work within a large radius $(R' \subset s^i_A)$, and work within a large radius $(R \subset s^j_B)$, denoted as $(\mathbf{\Gamma}(\gamma_\mathbf{R}, \gamma_{\mathbf{R}'}) = (\mathbf{tr}(\mathbf{R}), \mathbf{tr}(\mathbf{R}')) = (\mathbf{\Gamma_3}, \mathbf{\Gamma_4}); (\mathbf{\Gamma_1} = \mathbf{\Gamma_2} = \mathbf{0}))$ is a tensor of dimension $((m \times n) = (m \times m) \times (m \times n))$. The tensor is shown below:

$$\Gamma_2 = \begin{array}{cc}
 & \begin{array}{cccccc} & & \Gamma_1 & & & \Gamma_2 \\ 1 & \cdots & n & 1 & \cdots & m \end{array} \\
\begin{array}{c} 1 \\ \vdots \\ n \\ 1 \\ \vdots \\ m \end{array} & \left(\begin{array}{ccc|ccc}
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
\hline
tr(R')_{1,1} & \cdots & 0 & tr(R)_{1,1} & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & tr(R')_{m,n} & 0 & \cdots & tr(R)_{m,m}
\end{array}\right) \\
 & \begin{array}{cccccc} & & \Gamma_3 & & & \Gamma_4 \end{array}
\end{array}$$

3) $(\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{R}}))$ is an action tensor that combines work within a small radius $(r \subset s^i_A)$, and work within a large radius $(R \subset s^j_B)$, denoted as $(\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{R}}) = \mathbf{tr}(\mathbf{r}), \mathbf{tr}(\mathbf{R}) = (\mathbf{\Gamma_1}, \mathbf{\Gamma_4}); (\mathbf{\Gamma_2} = \mathbf{\Gamma_3} = \mathbf{0}))$. The final tensor is a tensor of dimension $((n+m) \times (n+m))$ with the first $(n \times n)$ diagonal entries being $(\mathbf{\Gamma_1})$, and the last diagonal entries being $(\mathbf{\Gamma_4})$. All other entries are zeros. The tensor is shown below:

$$\Gamma_3 = \begin{array}{cc}
 & \begin{array}{cccccc} & & \Gamma_1 & & & \Gamma_2 \\ 1 & \cdots & n & 1 & \cdots & m \end{array} \\
\begin{array}{c} 1 \\ \vdots \\ n \\ 1 \\ \vdots \\ m \end{array} & \left(\begin{array}{ccc|ccc}
tr(r)_{1,1} & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & t(r)_{n,n} & 0 & \cdots & 0 \\
\hline
0 & \cdots & 0 & tr(R)_{1,1} & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & tr(R)_{m,m}
\end{array}\right) \\
 & \begin{array}{cccccc} & & \Gamma_3 & & & \Gamma_4 \end{array}
\end{array}$$

4) $(\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{R'}}))$ is an action tensor that combines small work within a small radius $(r \subset s^i_A)$, and large work within a small radius $(R' \subset s^i_A)$, denoted as $(\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{R'}}) = \mathbf{tr}(\mathbf{r}), (\mathbf{tr}(\mathbf{R'})) = \mathbf{\Gamma_1}, (\mathbf{\Gamma_3}); (\mathbf{\Gamma_2} = \mathbf{\Gamma_4} = \mathbf{0}))$ is a tensor of dimension $((n+m) \times (n+m))$. The tensor is shown below:

$$\Gamma_4 = \begin{array}{cc}
 & \begin{array}{cccccc} & & \Gamma_1 & & & \Gamma_2 \\ 1 & \cdots & n & 1 & \cdots & m \end{array} \\
\begin{array}{c} 1 \\ \vdots \\ n \\ 1 \\ \vdots \\ m \end{array} & \left(\begin{array}{ccc|ccc}
tr(r)_{1,1} & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & t(r)_{n,n} & 0 & \cdots & 0 \\
\hline
tr(R')_{1,1} & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & tr(R')_{m,n} & 0 & \cdots & 0
\end{array}\right) \\
 & \begin{array}{cccccc} & & \Gamma_3 & & & \Gamma_4 \end{array}
\end{array}$$

5) $(\mathbf{\Gamma}(\gamma_{\mathbf{r'}}, \gamma_{\mathbf{R}}))$ is an action tensor that combines small work within a large radius $(r' \subset s^j_B)$, and large work within a large radius $(R \subset s^j_B)$, denoted as $(\mathbf{\Gamma}(\gamma_{\mathbf{r'}}, \gamma_{\mathbf{R}}) = \mathbf{tr}(\mathbf{r'}), \mathbf{tr}(\mathbf{R}) = (\mathbf{\Gamma_2}, \mathbf{\Gamma_4}); (\mathbf{\Gamma_1} = \mathbf{\Gamma_3} = \mathbf{0}))$ is a tensor of dimension $((n+m) \times (n+m))$. The tensor is shown below:

$$\Gamma_5 = \begin{array}{c|ccc|ccc}
 & & & \Gamma_1 & & & \Gamma_2 \\
 & 1 & \cdots & n & 1 & \cdots & m \\
1 & 0 & \cdots & 0 & tr(r')_{1,1} & \cdots & 0 \\
\vdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\
n & 0 & \cdots & 0 & 0 & \cdots & t(r')_{n,m} \\
\hline
1 & 0 & \cdots & 0 & tr(R)_{1,1} & \cdots & 0 \\
\vdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\
m & 0 & \cdots & 0 & 0 & \cdots & tr(R)_{m,m} \\
 & & & \Gamma_3 & & & \Gamma_4
\end{array}$$

6) $(\mathbf{\Gamma}(\gamma_{\mathbf{r}'}, \gamma_{\mathbf{R}'}))$ is an action tensor that combines work within a small radius $(r' \subset s_B^j)$, and work within a large radius $(R' \subset s_A^i)$, denoted as $(\mathbf{\Gamma}(\gamma_{\mathbf{r}'}, \gamma_{\mathbf{R}'}) = \mathbf{tr(r'), tr(R')} = \mathbf{\Gamma_2, \Gamma_3}; (\mathbf{\Gamma_1 = \Gamma_4 = 0}))$ is a tensor of dimension a tensor of dimension $((n+m) \times (n+m))$. The tensor is shown below:

$$\Gamma_6 = \begin{array}{c|ccc|ccc}
 & & & \Gamma_1 & & & \Gamma_2 \\
 & 1 & \cdots & n & 1 & \cdots & m \\
1 & 0 & \cdots & 0 & tr(r')_{1,1} & \cdots & 0 \\
\vdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\
n & 0 & \cdots & 0 & 0 & \cdots & t(r')_{n,m} \\
\hline
1 & tr(R')_{1,1} & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\
m & 0 & \cdots & tr(R')_{m,n} & 0 & \cdots & 0 \\
 & & & \Gamma_3 & & & \Gamma_4
\end{array}$$

7) $(\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{R}}) \cap \mathbf{\Gamma}(\gamma_{\mathbf{r}'}, \gamma_{\mathbf{R}'}))$ is an action tensor that is the intersection of tensor actions $(\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{R}}))$, and tensor action $(\mathbf{\Gamma}(\gamma_{\mathbf{r}'}, \gamma_{\mathbf{R}'}))$. The resulting tensor is $(\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{R}}) \cap \mathbf{\Gamma}(\gamma_{\mathbf{r}'}, \gamma_{\mathbf{R}'}) = (tr(r) \times tr(R')); (\mathbf{\Gamma_1 = \Gamma_4 = 0}))$ is a tensor of dimension a tensor of dimension $((n+m) \times (n+m))$. The tensor is shown below. In the block matrix, (t) denotes transpose.

$$\Gamma_7 = \begin{array}{c|cccccc}
 & 1 & \cdots & n & 1 & \cdots & m \\
1 & tr(r)_{1,1} \times (tr(R')_{1,1})^t & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\
n & 0 & \cdots & tr(r)_{n,n} \times (tr(R')_{m,n})^t & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & tr(R)_{1,1} & \cdots & 0 \\
\vdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\
m & 0 & \cdots & 0 & 0 & \cdots & tr(R)_{m,m}
\end{array}$$

8) Similarly, $(\mathbf{\Gamma}(\gamma_{\mathbf{r}'}, \gamma_{\mathbf{R}}) \cap \mathbf{\Gamma}(\gamma_{\mathbf{r}'}, \gamma_{\mathbf{R}'}))$ is an action tensor that is the intersection of tensor actions $(\mathbf{\Gamma}(\gamma_{\mathbf{r}'}, \gamma_{\mathbf{R}}))$, and tensor action $(\mathbf{\Gamma}(\gamma_{\mathbf{r}'}, \gamma_{\mathbf{R}'}))$. The resulting tensor is $(\mathbf{\Gamma}(\gamma_{\mathbf{r}'}, \gamma_{\mathbf{R}}) \cap \mathbf{\Gamma}(\gamma_{\mathbf{r}'}, \gamma_{\mathbf{R}'}) = (tr(R) \times tr(R')); (\mathbf{\Gamma_1 = \Gamma_3 = 0}))$ is a tensor of dimension a tensor of dimension $((n+m) \times (n+m))$. The tensor is shown below:

$$\Gamma_8 = \begin{array}{cc}
 & \begin{array}{ccc|ccc} & & \Gamma_1 & & & \Gamma_2 \\ 1 & \cdots & n & 1 & \cdots & m \end{array} \\
\begin{array}{c} 1 \\ \vdots \\ n \\ 1 \\ \vdots \\ m \end{array} & \left(\begin{array}{ccc|ccc}
0 & \cdots & 0 & tr(r')_{1,1} & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & t(r')_{n,m} \\ \hline
tr(R')_{1,1} & \cdots & 0 & tr(R)_{1,1} \times tr(R')_{1,1} & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & tr(R')_{m,n} & 0 & \cdots & tr(R)_{m,m} \times tr(R')_{m,n}
\end{array} \right) \\
 & \begin{array}{ccc|ccc} & & \Gamma_3 & & & \Gamma_4 \end{array}
\end{array}$$

9) $(\boldsymbol{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{R}}) \cup \boldsymbol{\Gamma}(\gamma_{\mathbf{r'}}, \gamma_{\mathbf{R'}}))$ is an action tensor that is the union of tensor actions $(\boldsymbol{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{R}}))$, and tensor action $(\boldsymbol{\Gamma}(\gamma_{\mathbf{r'}}, \gamma_{\mathbf{R'}}))$. The resulting tensor is $(\boldsymbol{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{R}}) \cup \boldsymbol{\Gamma}(\gamma_{\mathbf{r'}}, \gamma_{\mathbf{R'}}) = (\boldsymbol{\Gamma_1}, \boldsymbol{\Gamma_3}, \boldsymbol{\Gamma_4})(\boldsymbol{\Gamma_2} = \mathbf{0}))$, is a tensor of dimension a tensor of dimension $((n+m) \times (n+m))$. The tensor is shown below:

$$\Gamma_9 = \begin{array}{cc}
 & \begin{array}{ccc|ccc} & & \Gamma_1 & & & \Gamma_2 \\ 1 & \cdots & n & 1 & \cdots & m \end{array} \\
\begin{array}{c} 1 \\ \vdots \\ n \\ 1 \\ \vdots \\ m \end{array} & \left(\begin{array}{ccc|ccc}
tr(r)_{1,1} & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & t(r)_{n,n} & 0 & \cdots & 0 \\ \hline
tr(R')_{1,1} & \cdots & 0 & tr(R)_{1,1} & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & tr(R')_{m,n} & 0 & \cdots & tr(R)_{m,m}
\end{array} \right) \\
 & \begin{array}{ccc|ccc} & & \Gamma_3 & & & \Gamma_4 \end{array}
\end{array}$$

10) Similarly, $(\boldsymbol{\Gamma}(\gamma_{\mathbf{r'}}, \gamma_{\mathbf{R}}) \cup \boldsymbol{\Gamma}(\gamma_{\mathbf{r'}}, \gamma_{\mathbf{R'}}))$ is an action tensor that is the union of tensor actions $(\boldsymbol{\Gamma}(\gamma_{\mathbf{r'}}, \gamma_{\mathbf{R}}))$, and tensor action $(\boldsymbol{\Gamma}(\gamma_{\mathbf{r'}}, \gamma_{\mathbf{R'}}))$. The resulting tensor is $(\boldsymbol{\Gamma}(\gamma_{\mathbf{r'}}, \gamma_{\mathbf{R}}) \cup \boldsymbol{\Gamma}(\gamma_{\mathbf{r'}}, \gamma_{\mathbf{R'}}) = (\boldsymbol{\Gamma_2}, \boldsymbol{\Gamma_3}, \boldsymbol{\Gamma_4})(\boldsymbol{\Gamma_1} = \mathbf{0}))$, is a tensor of dimension a tensor of dimension $((n+m) \times (n+m))$. The tensor is shown below:

$$\Gamma_{10} = \begin{array}{cc}
 & \begin{array}{ccc|ccc} & & \Gamma_1 & & & \Gamma_2 \\ 1 & \cdots & n & 1 & \cdots & m \end{array} \\
\begin{array}{c} 1 \\ \vdots \\ n \\ 1 \\ \vdots \\ m \end{array} & \left(\begin{array}{ccc|ccc}
0 & \cdots & 0 & tr(r')_{1,1} & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & t(r')_{n,m} \\ \hline
tr(R')_{1,1} & \cdots & 0 & tr(R)_{1,1} & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & tr(R')_{m,n} & 0 & \cdots & tr(R)_{m,m}
\end{array} \right) \\
 & \begin{array}{ccc|ccc} & & \Gamma_3 & & & \Gamma_4 \end{array}
\end{array}$$

11) $(\boldsymbol{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{R}}) \cap \boldsymbol{\Gamma}(\gamma_{\mathbf{R}}, \gamma_{\mathbf{R'}}))$ is an action tensor that is the intersection of tensor actions $(\boldsymbol{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{R}}))$, and tensor action $(\boldsymbol{\Gamma}(\gamma_{\mathbf{R}}, \gamma_{\mathbf{R'}}))$. The resulting tensor is $(\boldsymbol{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{R}}) \cap \boldsymbol{\Gamma}(\gamma_{\mathbf{R}}, \gamma_{\mathbf{R'}}) = (\boldsymbol{\Gamma_4})(\boldsymbol{\Gamma_1}, \boldsymbol{\Gamma_2}, \boldsymbol{\Gamma_3}) = \mathbf{0}))$ is a tensor of dimension a tensor of dimension $((n+m) \times (n+m))$. The tensor is shown below. In the block matrix.

$$\Gamma_{11} = \begin{array}{cc} & \begin{array}{cccccc} & & \Gamma_1 & & & \Gamma_2 \end{array} \\ \begin{array}{c} \\ 1 \\ \vdots \\ n \\ 1 \\ \vdots \\ m \\ \\ \end{array} & \left(\begin{array}{ccc|ccc} 1 & \cdots & n & 1 & \cdots & m \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ \hline 0 & \cdots & 0 & tr(R)_{1,1} & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & tr(R)_{m,m} \\ & & \Gamma_3 & & & \Gamma_4 \end{array}\right) \end{array}$$

12) $(\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{R}}) \cap \frac{\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{R'}}}{\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{r'}})})$ is an action tensor that is the intersection of tensor actions $(\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{R}}))$, and tensor action $(\frac{\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{R'}})}{\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{r'}})})$. The tensor action, $(\frac{\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{R'}})}{\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{r'}})})$ is equal to an identity block matrix of dimension $((n+m) \times (n+m))$ shown as:

$$\begin{array}{cc} & \begin{array}{cccccc} & & \Gamma_1 & & & \Gamma_2 \end{array} \\ \begin{array}{c} \\ 1 \\ \vdots \\ n \\ 1 \\ \vdots \\ m \\ \\ \end{array} & \left(\begin{array}{ccc|ccc} 1 & \cdots & n & 1 & \cdots & m \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 1 & 0 & \cdots & 0 \\ \hline 0 & \cdots & 0 & 1 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 1 \\ & & \Gamma_3 & & & \Gamma_4 \end{array}\right) \end{array}$$

the intersection of block matrix for tensor action $(\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{R}}))$, and the identity block matrix above, gives the block matrix of dimension $((n+m) \times (n+m))$ shown below:

$$\Gamma_{12} = \begin{array}{cc} & \begin{array}{cccccc} & & \Gamma_1 & & & \Gamma_2 \end{array} \\ \begin{array}{c} \\ 1 \\ \vdots \\ n \\ 1 \\ \vdots \\ m \\ \\ \end{array} & \left(\begin{array}{ccc|ccc} 1 & \cdots & n & 1 & \cdots & m \\ tr(r)_{1,1} & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & t(r)_{n,n} & 0 & \cdots & 0 \\ \hline 0 & \cdots & 0 & tr(R)_{1,1} & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & tr(R)_{m,m} \\ & & \Gamma_3 & & & \Gamma_4 \end{array}\right) \end{array}$$

13) $(\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{R}}) \cap \mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{r'}}))$ is an action tensor that is the intersection of tensor actions $(\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{R}}))$, and tensor action $(\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{r'}}))$. The resulting tensor is $(\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{R}}) \cap \mathbf{\Gamma}(\gamma_{\mathbf{r'}}, \gamma_{\mathbf{R'}}) = (tr(R) \times tr(r')); (\mathbf{\Gamma_2} = \mathbf{\Gamma_3} = \mathbf{0}))$ is a tensor of dimension a tensor of dimension $((n+m) \times (n+m))$. The tensor is shown below. In the block matrix, (t) denotes transpose.

$$\Gamma_{13} = \begin{array}{cc}
 & \begin{array}{cccccc} & & \Gamma_1 & & & \Gamma_2 \\ 1 & \cdots & n & 1 & \cdots & m \end{array} \\
\begin{array}{c} 1 \\ \vdots \\ n \\ 1 \\ \vdots \\ m \end{array} & \left(\begin{array}{ccc|ccc}
tr(r)_{1,1} & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & t(r)_{n,n} & 0 & \cdots & 0 \\
\hline
0 & \cdots & 0 & tr(R)_{1,1} \times tr(r')^{t}_{1,1} & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & tr(R)_{m,m} \times tr(r')^{t}_{n,m}
\end{array}\right) \\
 & \begin{array}{cccccc} & & \Gamma_3 & & & \Gamma_4 \end{array}
\end{array}$$

14) $(\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{R}}) \cup \frac{\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{R'}}}{\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{r'}})})$ is an action tensor that is the intersection of tensor actions $(\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{R}}))$, and tensor action $(\frac{\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{R'}})}{\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{r'}})})$. The tensor action, $(\frac{\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{R'}})}{\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{r'}})})$ is equal to an identity block matrix of dimension $((n+m) \times (n+m))$ shown as:

$$\begin{array}{cc}
 & \begin{array}{cccccc} & & \Gamma_1 & & & \Gamma_2 \\ 1 & \cdots & n & 1 & \cdots & m \end{array} \\
\begin{array}{c} 1 \\ \vdots \\ n \\ 1 \\ \vdots \\ m \end{array} & \left(\begin{array}{ccc|ccc}
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 1 & 0 & \cdots & 0 \\
\hline
0 & \cdots & 0 & 1 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 1
\end{array}\right) \\
 & \begin{array}{cccccc} & & \Gamma_3 & & & \Gamma_4 \end{array}
\end{array}$$

the intersection of block matrix for tensor action $(\mathbf{\Gamma}(\gamma_{\mathbf{r}}, \gamma_{\mathbf{R}}))$, and the identity block matrix above, gives the block matrix of dimension $((n+m) \times (n+m))$ shown below:

$$\Gamma_{14} = \begin{array}{cc}
 & \begin{array}{cccccc} & & \Gamma_1 & & & \Gamma_2 \\ 1 & \cdots & n & 1 & \cdots & m \end{array} \\
\begin{array}{c} 1 \\ \vdots \\ n \\ 1 \\ \vdots \\ m \end{array} & \left(\begin{array}{ccc|ccc}
tr(r)_{1,1} + 1 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & t(r)_{n,n} + 1 & 0 & \cdots & 0 \\
\hline
0 & \cdots & 0 & tr(R)_{1,1} & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & tr(R)_{m,m}
\end{array}\right) \\
 & \begin{array}{cccccc} & & \Gamma_3 & & & \Gamma_4 \end{array}
\end{array}$$

Each of the 10 tensor action types correspond to one of the 14 action tensors that represent action. Depending on the syntax and syntax operators one of the action axis becomes the causal action that determines the quantity demanded, (q) and the price, (p).

4 Mathematical interpretation of the axioms of DCTA

Axiom 1. Causality

Each individual possesses a thought process represented by a collection of syntax. The syntax shows the intentions of the individual by indicating the physical movements considered in order to reach the desired intention. The causality between intention (syntax), and the ensuing physical movements is given as a morphism from syntax (ξ), to action ($\Gamma(\gamma_r, \gamma_R)$), ($\xi \longrightarrow \Gamma(\gamma_r, \gamma_R)$), where (γ_r) represents physical movement or work done within a circular zone of small radius (r), and (γ_R) represents physical movements or work done within a circular zone of large radius (R). The circular representation of work or physical movement is the simplest mathematical form of representing action. Since action always takes place in a 3 dimensional physical space, the circular zones are mainly represented as spheroids within the 3 orthogonal basis (I, v, F), namely, Intensity, velocity, and Frequency. All 3 basis are the functions of different objects contained in syntax (ξ). Any syntax, can be transformed or modified when pieces of additional syntax is attached to the basic version. The transformation is mathematically represented by operators denoted as (Δ). Thus for any one syntax many permutations and combinations are possible through the application of operators (Δ). The syntax can be represented as a tensor in an infinite dimensional Causal Conceptual Space ($\mathbb{S}$), that includes all permutations and combinations of any syntax. At this stage syntax can be represented as a tensor manifold, namely, a tensor hyperplane denoted as ($H(\xi, \Delta)$). The homeomorphism from the causal syntax to action is given as ($\xi \xrightarrow{S^3(I,v,F)} \Gamma(\gamma_r, \gamma_R)$) , where (S^3) represents the spheroids of causal action.. This is a homeomorphism from an infinite dimensional space to a (3) dimensional space, through mapping given by (S^3).

Axiom 2. Syntax: Syntax is the cause of action or physical movement.

Each syntax is characterized by its' causal sets, causal groups, and causal categories. Each individual thought process represented by a collection of syntax. Each syntax is characterized by it's causal sets, causal groups and causal categories. The word causal is used to imply that syntax is the cause of action. The topological representation is as follows: Let the Causal Conceptual Space ($\mathbb{S}$) be denoted as an infinite dimensional space, ($\mathbb{S}^\infty$). The ($\mathbb{S}$) contains all possible casual sets (L_A), and (L_B). It is assumed that the two major sets (L_A, L_B) are such that $L_A \in \mathbb{S}^\infty$, and $L_B \in \mathbb{S}^\infty$. Causal sets L_A contain all syntax element that help with the logic of a syntax but do not include adjectives, adverbs, and verbs. Causal sets L_B contain all adjectives, adverbs , and verbs, thus $L_A \notin L_B$. Each casual sets (L_A) includes all possible subsets ($s_A^i \in L_A, ; i = 1, \cdots, n$), and ($L_B$) includes all possible subsets ($s_B^j \in L_B, ; j = 1, \cdots, m$). Causal subsets belong to causal groups (G_{L_A}), and (G_{L_B}). Causal groups are topological groups that possess a family of sub-groups obtained through the following operations, ($\oplus, \ominus, \otimes, \circ$), that give sub-groups, ($(G_{L_A})_i$), and ($(G_{L_B})_j$). The causal sub-groups are obtained through operations applied to causal subsets ($s_A^i \in L_A, ; i = 1, \cdots, n$), and ($s_B^j \in L_B, ; j = 1, \cdots, m$). The following conditions hold for causal subsets: 1) ($L_A \cap L_B = 0$), and 2)($L_A \cup L_B \neq 0$). Thus, ($s_A^i \cap s_B^j = 0; i = 1, \cdots, n; j = 1, \cdots, m$), and ($s_A^i \cup s_B^j = 0; i = 1, \cdots, n; j = 1, \cdots, m$). Causal groups can be constructed through the application of operators. This expands the conceptual causal space, $(\mathbb{S}^\infty)$.

Each of the main 2 groups $((G_{L_A})_i)$, and $((G_{L_B})_j)$ possess categories denoted as $((C_{L_A})_i)$, and $((C_{L_B})_j)$. The topological causal categories can be established through the algebra of causal groups. These categories are causal by structure, and conceptual as they constitute a syntax . Examples of 4 types of causal categories are given as: 1)Category 1, denoted as $(C_1 = C_{L_A} \in G_{L_A})$, and $(C_1 = \mathbf{p}^i_{i'}\{s^i_A\})$, where $(\mathbf{p}^i_{i'})$ is the permutation of (i') causal subsets from a total of (i) causal subsets. Group 1, is built as a function of a subset of $(\{s^i_A\} \in L_A)$, where $(i = 1, \cdots, n)$ designates the number of possible subsets of any set (L_A). Category 2, is denoted as $(C_2 = C_{L_B})$, and $(C_2 = \mathbf{p}^j_{j'}\{s^j_B\})$, where $(\mathbf{p}^j_{j'})$ is the permutation of (j') causal subsets from a total of (j) causal subsets. Category 2, is built as a function of a subset of $(\{s^j_B\} \in L_B)$, where $(j = 1, \cdots, m)$ designates the number of possible subsets of any set (L_B). Causal categories (3), and (4), are constructed as $(C_3 = \mathbf{c}^i_{i'}p(s^i_A)\{s^i_A\})$, where $(\mathbf{c}^i_{i'})$ is the combination of (i') causal sets out of a total (i) causal subsets, and $(p(s^i_A))$ is the probability that such a combination exists, and $(C_4 = \mathbf{c}^j_{j'}p(s^j_B)\{s^j_B\})$. Groups (3), and (4), are probabilistic categories that represent the the syntax altered to match the perception of an individual. It is possible to obtain an algebra of categories, through the union, $(\cup)$, and the intersection, $(\cap)$ operations. A variety of syntax can be constructed by manipulating categories. Categories become tensor categories. Examples of such a syntax construction through the use of categories is given as $((C_1 \oplus C_2) \cup C'_1) = (C_1 \oplus C'_1) \cup C_2))$, and $((C_1 \oplus C_2) \cap C'_1) = (C_1 \oplus C'_1) \cap C_2))$. $(C'_.)$ is any alternative versions of categories belonging to sets (L_A), and (L_B). The infinite causal space, $(\mathbb{S}^\infty)$ is established given the existence of causal global sets, and causal subsets belonging to causal groups and causal categories. The casual space, $(\mathbb{S}^\infty)$, includes causal conceptual operators, and categories of operators.

The key to the existence of a homemorphism between syntax, and action, $(H(\xi, \Delta)$-$\xrightarrow{S^3(I,v,F)} \Gamma(\gamma_r, \gamma_R)$), is to connect intention to work. Intention is best defined through an example, consider, the difference between a form and a shape. A form is a natural occurrence, whereas a shape is a constructed occurrence. Therefore, a syntax is considered to have a form that is inherent in the syntax. A syntax is considered to have a shape when it is constructed in a way that it represents the intention of an individual, in such a way that the intention specifies, the goal of the individual, and such that achieving this goal, requires work on the part of the individual. This work is considered to be physical movements that end in spatial displacement either within a small zone (r), or a large zone (R). The following Theorem proves that intention is mapped onto action. The following formalizes the definition of intention in a syntax:

Definition 4.1. Given an infinite dimensional conceptual causal space,, $(\mathbb{S}^\infty)$, each category $(C_{L_A} \subset \mathbb{S}^\infty)$, and $(C_{L_B} \subset \mathbb{S}^\infty)$ can be mapped onto a 3 dimensional space, (I, v, F) through the element of intention denoted by $(\mathbb{I})$, embedded in either (C_{L_A}), or (C_{L_B}) categories. The shape of intention is represented by a spheroid, (S^3).

The following Theorem gives a proof that any causal conceptual hyperplane $(H(\xi, \Delta))$ that consists of a collection of categories, which means every point (ξ, Δ) on the causal hyperplane contains categories in both groups (G_{L_A}), and (G_{L_B}). It is shown that there exists homeomorphism that maps each category onto action through application of intention operators, $(\mathbb{I})$, represented as an operator that transforms the categories into work in a 3 dimensional space shown as spheroid shapes, that imply action tensors $(\Gamma(\gamma_r, \gamma_R))$

Theorem 4.1. *The homeomorphism from a syntax to an action,* $(H(\xi, \Delta) \xrightarrow{\Gamma} (\gamma_r, \gamma_R)$ *), is the result of the application of a tensor operator, called intention,* $(\mathbb{I})$*.* $(\mathbb{I})$ *is an special operator that maps any point on* $(H(\xi, \Delta))$*to spheroids that specify the from of an action. These actions are enumerated in the section on action.*

Proof. The homeomorphism $(H(\xi, \Delta) \xrightarrow{S^3(I,v,F)} \Gamma(\gamma_r, \gamma_R)$)is the generalization of the mapping of all categories to action. The intention operator,$(\mathbb{I})$ is the block matrix $(\mathbb{I} = \Gamma(\boldsymbol{.}))$ already introduced in section on action. By definition 4.1 the syntax (ξ, Δ) containing any categories (C_{L_A}), and (C_{L_B}), is a cohomology depending on $(\mathbb{I})$. Action, $(\Gamma(\gamma_r, \gamma_R))$, is the cohomology of the categories constructed by the operator, $(\mathbb{I})$ that contains the spheroids, $(S^3(I, v, F))$. The diagram in Figure 18 shows this process. In Figure 18, the categories $(C'_{L_A} \subset G_{L_A})$, and $(C'_{L_B} \subset G_{L_B})$ are any categories different from (C_{L_A}), and (C_{L_B}). The intention operator maps all categories to action through the application of the intention operator, $(\mathbb{I})$, in the geometrical form of spheroids.

The intention operator works in two stages. In stage (1), it identifies the elements of intention in the categories of (ξ, Δ). This stage produces the tensor $(\mathbb{I}_C)$, where (C) stands for category shown below:

$$
\begin{array}{cc}
 & \begin{array}{cccccc} & & \mathbb{I}_C^1 & & & \mathbb{I}_C^2 \\ 1 & \cdots & n & 1 & \cdots & m \end{array} \\
\begin{array}{c} 1 \\ \vdots \\ n \\ 1 \\ \vdots \\ m \end{array} &
\left(\begin{array}{ccc|ccc}
((\{C_A^*\})^r)_{1,1} & \cdots & 0 & ((\{C_B^*\})^{r'})_{1,1} & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & ((\{C_A^*\})^r)_{n,n} & 0 & \cdots & ((\{C_B^*\})^{r'})_{n,m} \\
\hline
((\{C_A^*\})^{R'})_{1,1} & \cdots & 0 & ((\{C_B^*\})^{R'})_{1,1} & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & ((\{C_A^*\})^R)_{m,n} & 0 & \cdots & ((\{C_B^*\})^R)_{m,m}
\end{array}\right) \\
 & \begin{array}{cccccc} & & \mathbb{I}_C^3 & & & \mathbb{I}_C^4 \end{array}
\end{array}
$$

In the operator $(\mathbb{I}_C)$ is a block diagonal matrix of size $((n+m) \times (n+m))$. There are (4) block matrices, $(\mathbb{I}_C = \mathbb{I}_C^1, \mathbb{I}_C^2, \mathbb{I}_C^3, \mathbb{I}_C^4)$. The block, $(\mathbb{I}_C^1)$ is a diagonal matrix of size $(n \times n)$. The diagonal entries are causal sets that contain intention. Let's denote these sets as $((\{C_A^*\})^r)_{i,j} \subset C_{L_A}; i = j; i = 1, \cdots, n)$. The diagonal elements are intention verbs that indicate action within a zone of radius (γ_r). The second block,$(\mathbb{I}_C^2)$ is a diagonal matrix of size $(n \times m)$. The diagonal elements are causal sets that contain intention. Let's denote these sets as $((\{C_B^*\})^{r'})_{i,j} \subset C_{L_B}; i = 1, \cdots, n; j = 1, \cdots, m)$. The diagonal elements are intention verbs that indicate small movement actions within a zone of radius (R), $(\gamma_{r'} \subset \gamma_R)$. Block $(\mathbb{I}_C^3)$ is a diagonal matrix of size

$(m \times n)$. The diagonal entries are causal sets that contain intention. Let's denote these sets as $(((\{C_A^*\})^{R'})_{i,j} \subset C_{L_A}; i = 1, \cdots, m; j = 1, \cdots, n)$. The diagonal elements are intention verbs that indicate large movement actions within a zone of radius (r), $(\gamma_{R'} \subset \gamma_r)$. Block $(\mathbb{I}_C^4)$ is a diagonal matrix of size $(m \times m)$. The diagonal entries are causal sets that contain intention. Let's denote these sets as $(((\{C_B^*\})^R)_{i,j} \subset C_{L_B}; i = jj = 1, \cdots, m)$. The diagonal elements are intention verbs that indicate large movement actions within a zone of radius (R),(γ_R). In stage (2), the interim operator, $(\mathbb{I}_C)$ which is the intermediate tensor is mapped onto the intention tensor, $(\mathbb{I} = \mathbb{I}_C \to \Gamma(\centerdot))$.

$$
\begin{array}{cc}
 & \begin{array}{cccccc} & & \mathbb{I}_C^1 & & & \mathbb{I}_C^2 \\ 1 & \cdots & n & 1 & \cdots & m \end{array} \\
\begin{array}{c} 1 \\ \vdots \\ n \\ 1 \\ \vdots \\ m \end{array} &
\left(\begin{array}{ccc|ccc}
((\{C_A^*\})^r)_{1,1} & \cdots & 0 & ((\{C_B^*\})^{r'})_{1,1} & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & ((\{C_A^*\})^r)_{n,n} & 0 & \cdots & ((\{C_B^*\})^{r'})_{n,m} \\
\hline
((\{C_A^*\})^{R'})_{1,1} & \cdots & 0 & ((\{C_B^*\})^R)_{1,1} & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & ((\{C_A^*\})^{R'})_{m,n} & 0 & \cdots & ((\{C_B^*\})^R)_{m,m}
\end{array}\right) \\
 & \begin{array}{cccccc} & & \mathbb{I}_C^3 & & & \mathbb{I}_C^4 \end{array}
\end{array}
$$

$$
\to \Gamma(\centerdot) =
\begin{array}{cc}
 & \begin{array}{cccccc} & & \Gamma_1 & & & \Gamma_2 \\ 1 & \cdots & n & 1 & \cdots & m \end{array} \\
\begin{array}{c} 1 \\ \vdots \\ n \\ 1 \\ \vdots \\ m \end{array} &
\left(\begin{array}{ccc|ccc}
tr(r)_{1,1} & \cdots & 0 & tr(r')_{1,1} & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & t(r)_{n,n} & 0 & \cdots & t(r')_{n,m} \\
\hline
tr(R')_{1,1} & \cdots & 0 & tr(R)_{1,1} & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & tr(R')_{m,n} & 0 & \cdots & tr(R)_{m,m}
\end{array}\right) \\
 & \begin{array}{cccccc} & & \Gamma_3 & & & \Gamma_4 \end{array}
\end{array}
$$

□

$$
\begin{tikzcd}
(\xi, \Delta) \arrow[r, dash, "\Delta"] \arrow[d, dash, "(\Delta)"'] & (C'_{L_A}, C'_{L_B}) \arrow[d, dash, "\mathbb{I} = S^3(I, v, F)"] \\
(C_{L_A}, C_{L_B}) \arrow[r, dash, "\mathbb{I} = S^3(I, v, F)"'] & \Gamma(\gamma_r, \gamma_R)
\end{tikzcd}
$$

Figure 18. Cohomology of syntax, and action

Based on the Theorem 4.1, adding (Δ) to the basic syntax (ξ) allows for the inclusion of intention in the modified syntax, (ξ, Δ), $(\xi \xrightarrow{(} \xi, \Delta))$, and maps it into different categories, (C_{L_A}),(C_{L_B}), and (C'_{L_A}),(C'_{L_B}), where $(C_{L_A} \neq C'_{L_A})$, and $(C_{L_B} \neq C'_{L_B})$. Intention operator maps these categories onto different types of action, $(\Gamma(\gamma_r, \gamma_R))$, that are represented as 3 dimensional spheroids in an Euclidean space. Each category is mapped onto one of the (12) action types,$(\Gamma(\boldsymbol{.}) = \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5, \Gamma_1, \Gamma_7, \Gamma_8, \Gamma_9, \Gamma_{10}$,-$\Gamma_{11}, \Gamma_{12})$ that depend on intention tensor operator,$(\mathbb{I})$, introduced in the section on action. There are two types of operators, the transformation operator, (Δ) that exists in the conceptual space (S^∞), and the intention operators that exist as a mapping from the conceptual space to a 3D space, $(S^\infty \rightarrow \Re^3)$ space. The transformation operator has many sub operators all tensors in the tensor transformation operator space, (S^∞). The function of any operator, (Δ), is to transform a syntax, to a transformed syntax $(\xi \rightarrow (\xi, \Delta))$ is in several distinct ways. 1)(Δ), changes the dialectic of a syntax by enhancing the logic, or clarifying the logic or adding extra elements.

In all these cases, additions of any kind, translate to the expansion of groups already introduced, (G_{L_A}),(G_{L_B}), meaning extra sub-groups, $(s^i_{L_A}; i = 1, \cdots, I, I + 1, \cdots)$, and $(s^j_{L_B}; j = 1, \cdots, J, J + 1, \cdots)$. 2) Transformation (Δ), adds extra groups (G_{L_C}) where $(G_{L_C} \subset S^\infty)$. 3)Transformation (Δ), expands the categories within each group by adding extra elements to $(C^{ii}_{L_A} \subset G_{L_A}; ii = 1, \cdots, II, II + 1, \cdots)$, and $(C^{jj}_{L_B} \subset G_{L_B}; jj = 1, \cdots, JJ, JJ + 1, \cdots)$. 4) The intention operator,$(\mathbb{I})$,provides a mapping that specifies the embedded intention in a specific geometrical spheroid shapes, given any category (C). An example, for group (G_{L_A}) of the impact of the conceptual operator is shown in Figure (18+).

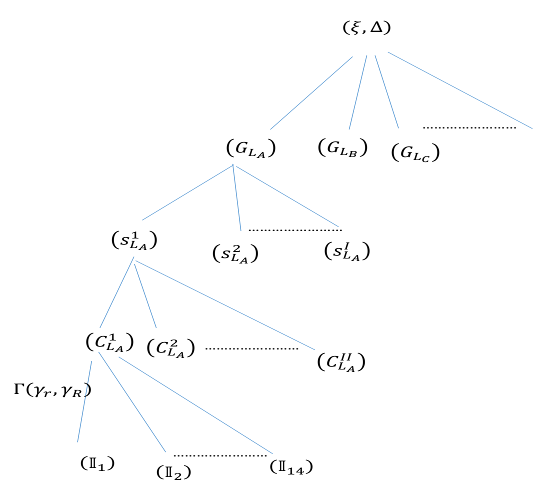


Figure 18+. The impact of operators on groups, sub-groups, and categories

Axiom 3. Evolution of action: is the Biological evolution towards action. As syntax evolves, it causes more complicated physical movements in space, in other words, evolved actions.

In other words, as syntax is transformed through the use of the transformation operator (Δ), it assumes various interpretations that are different from the initial interpretation of the basic syntax, (syntax without any transformations (ξ). Once syntax is modified, it induces various permutations, combinations, fractions, and probabilistic actions that is represented as a tensor of interaction between $((\gamma_r)_k)$, and $((\gamma_R)_k)$ for $(k = 1, 2)$. $((\gamma_r)_1)$is small movement actions in a zone of radius (r), and $((\gamma_r)_2 = \gamma_{r'})$ is small movement actions within a zone of radius (R). Similarly $((\gamma_R)_1)$ is large movement actions within a zone of radius (r), and $((\gamma_R)_2 = \gamma_{R'})$ is large movement actions within a zone of radius (R). Therefore, evolution can be formulated as functions of (permutations, combinations, fractions, and probabilities). Thus, evolution is represented as an operator, $(\mathfrak{I}_{\mathfrak{E}})$, where $(\mathfrak{I}_{\mathfrak{E}})$ represents a diagonal block matrix operator, of size $((n + m) \times (n + m))$. There are (4) types of evolution operators; $(\mathfrak{I}^{\mathfrak{p}}_{\mathfrak{E}}; \mathfrak{I}^{\mathfrak{c}}_{\mathfrak{E}}; \mathfrak{I}^{\beta}_{\mathfrak{E}}; \mathfrak{I}^{\mathfrak{pr}}_{\mathfrak{E}})$. $(\mathfrak{I}^{\mathfrak{p}}_{\mathfrak{E}})$, is an evolution operator, of size $((n+m) \times (n+m))$ with diagonal entries equal to permutations on the elements of causal categories (C^i_A), and (C^j_B).

$(\mathfrak{I}^{\mathfrak{c}}_{\mathfrak{E}})$ is an evolution operator, of size $((n + m) \times (n + m))$ with diagonal entries equal to combinations of the elements of causal categories (C^i_A), and (C^j_B) that determine $((\gamma_r)_k)$, and $((\gamma_R)_k)$ for $(k = 1, 2)$. $(\mathfrak{I}^{\beta}_{\mathfrak{E}})$ is an evolution operator, of size $((n+m) \times (n+m))$ with diagonal entries equal to fractions of the elements of causal categories (C^i_A), and (C^j_B) that determine $((\gamma_r)_k)$, and $((\gamma_R)_k)$ for $(k = 1, 2)$. $(\mathfrak{I}^{\mathfrak{pr}}_{\mathfrak{E}})$ is an evolution operator, of size $((n + m) \times (n + m))$ with diagonal entries equal to probabilities of the elements of causal categories (C^i_A), and (C^j_B) that determine $((\gamma_r)_k)$, and $((\gamma_R)_k)$ for $(k = 1, 2)$. The permutations,combinations, fractions, and probabilities mathematical formulations are explained in an earlier section on action.

There are (2) stages in the process of the evolution of action. In stage (1), the evolution operator, $(\mathfrak{I}_{\mathfrak{E}}{}^*)$ is multiplied by the interim operator, $(\mathbb{I}_C)$, $(\mathfrak{I}_{\mathfrak{E}}{}^* \otimes \mathbb{I}_C)$, where where $(*)$ stands for (permutations,combinations, fractions, and probabilities) operations. Let's denote the result of the tensor multiplication as $(\mathbb{I}^*_e)$. $(\mathbb{I}^*_e)$ is the modified interim operator of size $((n + m) \times (n + m))$. In stage (2), the modified interim operator, $(\mathbb{I}^*_e)$ is multiplied by the intention operator, $(\mathbb{I})$, as$(\mathbb{I}^*_e \otimes \mathbb{I} = \mathbb{I}^* = \Gamma(\centerdot)^*)$. The result of the multiplication is denoted as the modified intention operator,of size $((n + m) \times (n + m))$, $(\mathbb{I}^*)$. Given the (4) evolution operators, $(\mathfrak{I}^{\mathfrak{p}}_{\mathfrak{E}}; \mathfrak{I}^{\mathfrak{c}}_{\mathfrak{E}}; \mathfrak{I}^{\beta}_{\mathfrak{E}}; \mathfrak{I}^{\mathfrak{p}}_{\mathfrak{E}})$, there are (4) modified intention operators as follows: 1) $(\mathfrak{I}_{\mathfrak{E}}{}^p \otimes \mathbb{I}_C = \mathbb{I}^p_e)$, and $(\mathbb{I}^p_e \otimes \mathbb{I} = \mathbb{I}^p = \Gamma(\centerdot)^p)$. 2) $(\mathfrak{I}_{\mathfrak{E}}{}^c \otimes \mathbb{I}_C = \mathbb{I}^c_e)$, and $(\mathbb{I}^c_e \otimes \mathbb{I} = \mathbb{I}^c = \Gamma(\centerdot)^c)$. 3) $(\mathfrak{I}_{\mathfrak{E}}{}^\beta \otimes \mathbb{I}_C = \mathbb{I}^\beta_e)$, and $(\mathbb{I}^\beta_e \otimes \mathbb{I} = \mathbb{I}^\beta = \Gamma(\centerdot)^\beta)$, 4) $(\mathfrak{I}_{\mathfrak{E}}{}^{\mathfrak{pr}} \otimes \mathbb{I}_C = \mathbb{I}^{\mathfrak{pr}}_e)$, and $(\mathbb{I}^{\mathfrak{pr}}_e \otimes \mathbb{I} = \mathbb{I}^{\mathfrak{pr}} = \Gamma(\centerdot)^{\mathfrak{pr}})$. Any of the (4) modified intention operators correspond to any of the (12) action tensors.

Axiom 4. Dialectic as a source of syntax evolution.

Dialectic in this context refers to the interaction among evolution tensor operators, $(\mathfrak{I}^{\mathfrak{p}}_{\mathfrak{E}};\mathfrak{I}^{\mathfrak{c}}_{\mathfrak{E}};\mathfrak{I}^{\beta}_{\mathfrak{E}};\mathfrak{I}^{\mathfrak{pr}}_{\mathfrak{E}})$. Interaction refers to operations such as $(\oplus,\ominus,\otimes,\circ)$. $(\circ)$ is a symbol for composition. An example of such an interaction among evolution tensor operators, is $(\mathfrak{I}^{\beta}_{\mathfrak{E}}\circ\mathfrak{I}^{\mathfrak{c}}_{\mathfrak{E}})$ gives an evolution tensor operator, with diagonal entries where (β) is defined as a function of combination (c). The modified evolution operators introduce alternative perceptions of the logic of a syntax. Therefore dialectic is change in the logic of a syntax. The change in the logic of a syntax, is mapped onto action. This is done through interaction of dialectic operators, $(\mathfrak{I}^{*}_{\mathfrak{E}})$, where (*) stands for any of the variety (p,c,β,ρ), and the mathematical operations, $(\oplus,\ominus,\otimes,\circ)$, and the intention operators, interim (intermediate) intention operator, $(\mathbb{I}_C)$, and the intention operator, $(\mathbb{I})$. This is done in two stages. In stage (1), the dialectic operator is multiplied by the interim operator, using the example of the dialectic operator, for demonstration, $(\mathfrak{I}^{\beta}_{\mathfrak{E}}\circ\mathfrak{I}^{\mathfrak{c}}_{\mathfrak{E}}\otimes\mathbb{I}_C=\mathbb{I}^{\beta,c})$, produces the dialectic interim operator, $(\mathbb{I}^{\beta,c})$. In stage (2), the dialectic interim operator, $(\mathbb{I}^{\beta,c})$ is multiplied by the intention operator, $(\mathbb{I}^{\beta,c}\otimes\mathbb{I}=\Gamma(\centerdot)^{\beta,c})$. The outcome is an action operator $(\Gamma(\centerdot)^{\beta,c})$ that reflects the dialectic impact on a syntax. The action operator is a combination of some of the (12) types of actions already introduced. The dialectic process for the other (3) evolution tensor operators $(\mathfrak{I}^{\mathfrak{p}}_{\mathfrak{E}};\mathfrak{I}^{\mathfrak{c}}_{\mathfrak{E}};\mathfrak{I}^{\mathfrak{pr}}_{\mathfrak{E}})$ is similar to the example given, $(\mathbb{I}^{\mathfrak{p},c}\otimes\mathbb{I}=\Gamma(\centerdot)^{\mathfrak{p},c})$; $(\mathbb{I}^{\mathfrak{c},c}\otimes\mathbb{I}=\Gamma(\centerdot)^{\mathfrak{c},c})$; $(\mathbb{I}^{\mathfrak{pr},c}\otimes\mathbb{I}=\Gamma(\centerdot)^{\mathfrak{pr},c})$.

Axiom 5. Dialectic equilibrium

Dialectic is defined as variations in the type and the number of intention verbs in a syntax. Dialectic changes the perception and thus the issuing actions taken by an individual. Dialectic equilibrium is used to point to the limits of the application of dialectic in a syntax. The limit is reached when the impact of dialectic elements nullify the intention verbs. Therefore, dialectic equilibrium is reached when the intention operator is equivalent to a null tensor, $(\mathbb{I}=\mathbf{0})$. In the intermediate intention operator $(\mathbb{I}_C\neq\mathbf{0})$, the intention verbs in the categories cancel each other due to the effect of the dialectic. Given that dialectic application has a limit, it means that before reaching this limit the dialectic effect creates an optimal outcome. This means that the geometrical shapes created by the new movements retain a new regularity in shape. Conceptual hyperplanes, $(H(\xi,\Delta)\subset S^{\infty})$, are shapes without discontinuities, and the mapping onto action spheroids retain regularity in shape. Non-regular shapes are those with discontinuities. The dialectic equilibrium at its' optimal point is defined as transformations that maintain the homeomorphism, (more precisely the cohomology) of syntax to action, as is demonstrated in Figure 18. At the dialectic equilibrium at its' optimal point, the spheroids turn into spheres showing that actions are not distorted. Therefore, if actions are regular, the ultimate outcome is equilibrium between the DCTA demand and supply,$(q_D=f_D(\Gamma(\gamma_r,\gamma_R)))$, where (q_D) is the quantity demanded is a function of action, $(\Gamma(\gamma_r,\gamma_R))$, and the quantity supplied,$(q_S=f_S(\Gamma(\gamma_r,\gamma_R)))$ where (q_S) is the quantity supplied, is a function of action, $(\Gamma(\gamma_r,\gamma_R))$ are equal, $(q_D=q_S)$, given that the price for the quantity demanded, and the quantity supplied are equivalent,$(p_D(\Gamma(\gamma_r,\gamma_R))$, and $(p_S(\Gamma(\gamma_r,\gamma_R)))$, where (p_D) is the price of quantity demanded, and (p_S) is the price of quantity supplied.

The price axis, axis (2) on the action cuboid is a function of action, $(\Gamma(\gamma_r, \gamma_R))$.

Axiom 6. Forms of action: This axiom states that there are different types of actions. Different types of actions are micro actions, macro actions, mixed actions, proportional actions, and probabilistic actions. These refer to the (12) action tensors, $(\mathbf{\Gamma(.)})$ which is the generic action tensor that includes all action types. The (12) action tensors, $(\mathbf{\Gamma_1}, \mathbf{\Gamma_2}, \cdots, \mathbf{\Gamma_{12}})$, are either micro, or macro actions, or variations of the (2) types of actions. The forms of action refer to the nature of work that constitutes action. The nature of work is as follows:

1) Micro actions are the result of small movements or (work), within a zone of radius (r), and are denoted as (γ_r). Small movements or (work) in a zone of radius $(R > r)$,are denoted by $(\gamma_{r'})$, where $(r' \leq r)$. The micro actions are denoted as $(\Gamma(\gamma_r, \gamma_{r'}))$. Given that a syntax $(\xi, \Delta, \mathbb{I})$ that is represented as a basic segment, (ξ), that contains an initial logic, and the syntax transformation operator, (Δ), and the intention operator, $(\mathbb{I})$can break down into the basic logic that contains two main groups (G_{L_A}), and (G_{L_B}). The addition of the transformation operator,(Δ) introduces sub-groups, $(s^i_A \subset G_{L_A})$, and $(s^j_B \subset G_{L_B})$, and categories in each sub-group, $(C^i_A \subset G_{L_A})$, and $(C^j_B \subset G_{L_B})$. The intermediate intention operator, $(\mathbb{I}_C)$ gives the intention verbs. The intention verbs, are mapped onto the appropriate actions. In the case of micro actions. Both (γ_r), and $(\gamma_{r'})$ are the mapping of intention verbs, in both categories (C^i_A), and (C^j_B) representing a complicated mix of small movements in two different environments with small radius (r), and large radius (R).

2) Macro-actions are the result of large movements or (work), within a zone of radius (R), and are denoted as (γ_R). Large movements or (work) in a zone of radius (r),are denoted by $(\gamma_{R'})$, where $(R' < R)$. The macro actions are denoted as $(\Gamma(\gamma_R, \gamma_{R'}))$. Given a syntax $(\xi, \Delta, \mathbb{I})$, both (γ_R), and $(\gamma_{R'})$ are the mapping of intention verbs, in both categories (C^i_A), and (C^j_B) representing a complicated mix of large movements in two different environments with large radius (R), and small radius (r). Mixed actions, proportional actions, and probabilistic actions are variations of actions that mix, the two action types, $(\Gamma(\gamma_r, \gamma_{r'}))$, and $(\Gamma(\gamma_R, \gamma_{R'}))$. Mixed actions are given by $(\gamma_r, \gamma_{R'})$, (γ_r, γ_R), $(\gamma_r, \gamma_{r'})$, $(\gamma_{r'}, \gamma_{R'})$, $(\gamma_{r'}, \gamma_R)$, and $(\gamma_R, \gamma_{R'})$. proportional actions assigns weights to both micro, and macro, and mixed actions, and probabilistic actions, give each micro, macro, mixed and proportional actions, with a corresponding probabilities.

Probabilistic actions refer to the occurrence of action type given a syntax . This is formulated as the probability of an action given a syntax, $(p(\Gamma(\gamma^k_r, \gamma^k_R))) = \frac{p(\Gamma(\gamma^k_r, \gamma^k_R)) \cap (\xi, \Delta))}{p(\xi, \Delta)} = \frac{p(\Gamma(\gamma^k_r, \gamma^k_R)) . p(\xi, \Delta)}{p(\xi, \Delta)}; k = 1, 2)$ given that the probabilities are independent. The denominator $(p(\xi, \Delta) = (\mathfrak{I})^\rho_E \otimes \Delta)$ is equal to the multiplication of the tensor probability Evolution operator, $(\mathfrak{I})^\rho_E)$ and the tensor (Δ) is an $((n+m) \times (n+m))$ diagonal matrix with diagonal entries equal to the multiplication of the diagonal entries of the two tensors. The numerator $(p(\Gamma(\gamma^k_r, \gamma^k_R)) \,.\, p(\xi, \Delta))$ is an $((n+m) \times (n+m))$ diagonal matrix with diagonal entries equal to the trace of

(r^k), and the trace of (R^k). and the ($p(\xi, \Delta)$) matrix. The final probability matrix is an ($(n+m) \times (n+m)$) matrix with diagonal entries equal to the numerator divided by the denominator.

All action types are mapped onto (3) dimensional spheroids. For example, (γ_r, γ_R), is represented by an spheroid, with the following characteristics: 1) ($\sum \gamma_r > \sum \gamma_R$), this implies that the totality of work within zones of small radii, (r) is greater than the totality of work within zones of large radii (R), implying more micro actions, than macro actions. This produces stretched out or oblate spheroids. 2) in the case ($\sum \gamma_r < \sum \gamma_R$), the sum of work within zones of small radii (r), is smaller than the sum of work done within zones of large radii (R), implying more macro actions, than micro actions. This produces elongated or prolate spheroids. This is shown in Figure 19 below.

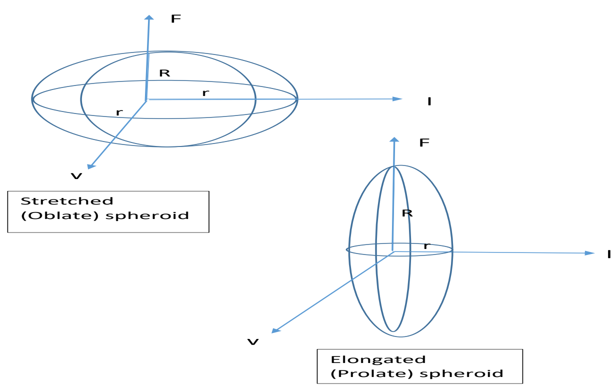


Figure 19. Stretched and Elongated Spheroids

Axiom 7. Transitional actions: micro actions transition to macro actions; Mixed actions are (micro-macro) actions can transition to other forms of action which are proportional or probabilistic. Syntax transition occurs due to the application of operators, (Δ), ($\mathbb{I}_C$), and ($\mathbb{I}$). At the stage of syntax transformation, due to the operators, (Δ), ($\mathbb{I}_C$), the geometrical manifestation, is the hyperplane, ($H(\xi, \Delta)$). The evolution, and the dialectic operators change the shape of the hyperplane. The mapping from any modified hyperplane to action through the intention operator, ($\mathbb{I}$) is geometrically, represented, as distorted spheroids. A graphical example is given in Figure 20. In Figure 20, the two modified hyperplanes, are denoted as ($H^1(\xi, \Delta)$), and ($H^2(\xi, \Delta)$). The modification of the syntax hyperplanes are due to the application of transformation operator, (Δ), and the intermediate intention operator, ($\mathbb{I}_C$). Each modified hyperplane is mapped to a modified spheroid.

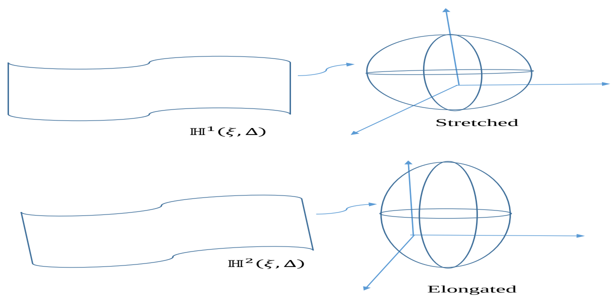


Figure 20. Spheroid shape modification due to syntax transformation

In Figure 20, the first hyperplane, $(H^1(\xi,\Delta))$, is mapped to a 3 dimensional stretched spheroid due to a syntax transformation. $(H^2(\xi,\Delta))$ is the second hyperplane, which is mapped to a 3 dimensional elongated spheroid due to a different syntax transformation. Stretched and elongated tensors are given as diagonal tensors of size $((n+m)\times(n+m))$. The diagonal entries are the trace of (r), trace of (r'), trace(R'), and trace (R), as is shown in the action tensor below. The trace is calculated given the equation of an spheroid given as $(\frac{I^2}{a_r^2}+\frac{v^2}{b_r^2}+\frac{F^2}{c_r^2}=r^2)$, where (r) stands for a generic radius which means that (r) can be replaced by the 3 other radius types , namely, (r'),(R'), and (R). The spheroid equation can be written in a matrix format given as follows:

$$
\begin{array}{cc}
 & \begin{array}{ccc} I & v & F \end{array} \\
\begin{array}{c} I \\ v \\ F \end{array} &
\begin{pmatrix}
(\frac{1}{a_r^2})_{1,1} & (\frac{1}{a_r^2})_{1,2} & (\frac{1}{a_r^2})_{1,3} \\
(\frac{1}{b_r^2})_{2,1} & (\frac{1}{b_r^2})_{2,2} & (\frac{1}{b_r^2})_{2,3} \\
(\frac{1}{c_r^2})_{3,1} & (\frac{1}{c_r^2})_{3,2} & (\frac{1}{c_r^2})_{3,3}
\end{pmatrix}
\otimes
\begin{pmatrix} I_r^2 \\ v_r^2 \\ F_r^2 \end{pmatrix}
\end{array}
$$

$$
= \begin{pmatrix} r_1^2 \\ r_2^2 \\ r_3^2 \end{pmatrix}
$$

Similar matrix expressions apply to the other 3 types of radii (r',R',R). Let the matrix of parameters be denoted by (A_1), the column vector of the axis by (B_1), and the column vector of the radii by (C_1). The matrix presentation for the other radii is then expressed as: for (r'), $(A_2;B_2;C_2)$, for (R'), $(A_3;B_3;C_3)$, and for (R), $(A_4;B_4;C_4)$. The traces of the 4 spheroid types based on the radii is given as, $(tr(A_1)=\frac{1}{a_r^2}+\frac{1}{b_r^2}+\frac{1}{c_r^2})$; similarly, $(tr(A_2)=\frac{1}{a_{r'}^2}+\frac{1}{b_{r'}^2}+\frac{1}{c_{r'}^2})$, $(tr(A_3)=\frac{1}{a_{R'}^2}+\frac{1}{b_{R'}^2}+\frac{1}{c_{R'}^2})$, and $(tr(A_4)=\frac{1}{a_R^2}+\frac{1}{b_R^2}+\frac{1}{c_R^2})$. In the case of a stretched, and elongated spheroids, where $(a=b)$, and $(a>c)$, and $(a<c)$, respectively, the traces are written putting $(a=b)$. Given the spheroid matrix equation, the action tensor given $(n=m)$ is then represented as:

$$\begin{array}{c} \\ 1 \\ \vdots \\ n \\ 1 \\ \vdots \\ m \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & \cdots & n & 1 & \cdots & m \end{array} \\ \begin{pmatrix} (A_1)_{1,1} & \cdots & 0 & (tr(A_2))_{1,1} & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & (tr(A_1)_{n,n} & 0 & \cdots & (tr(A_2))_{n,m} \\ (tr(A_3))_{1,1} & \cdots & 0 & tr(A_4)_{1,1} & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & (tr(A_3))_{m,n} & 0 & \cdots & tr(A_4)_{m,m} \end{pmatrix} \end{array}$$

Axiom 8. Interactions causing intra-actions. Interaction is a reference to the topology of action, (the manner of interactions). Interaction is a combined algebraic form of the interaction among action types, that defines the geometrical forms of action-interaction. Intra-action is defined as the extended consequence of interaction among different action types. For example, a micro-macro interaction of an individual that results in interacting with other individuals, and is extended into interacting with groups of individuals within (immediate and extended) the existing physical environment shared by all.Thus the topological aspect of the interaction and intra-action are expressed in functional terms.

This refers to the topology of action. Mainly, it is the actions of an individual with respect to the others, or the environmental obstacles or objects. The individual action is represented as $(\Gamma(\gamma_{r_k}, \gamma_{R_k}); k = 1, 2)$. Let $(\Gamma(\gamma_{r_{k'}}, \gamma_{R_{k'}}); k' \neq k)$ be the modified action of the individual with others and his modified actions dealing with instruments or objects in the environment. The existing topology between the two types of actions can be described either as a union of the two actions which is equivalent to a tensor addition $((\Gamma(\gamma_{r_k}, \gamma_{R_k}) \cup (\Gamma(\gamma_{r_{k'}}, \gamma_{R_{k'}}))$, $((\Gamma(\gamma_{r_k}, \gamma_{R_k}) \oplus (\Gamma(\gamma_{r_{k'}}, \gamma_{R_{k'}}))$, type(A) action, or the intersection of the two actions which is equivalent to a tensor multiplication, $((\Gamma(\gamma_{r_k}, \gamma_{R_k}) \cap (\Gamma(\gamma_{r_{k'}}, \gamma_{R_{k'}}))$,$((\Gamma(\gamma_{r_k}, \gamma_{R_k}) \otimes (\Gamma(\gamma_{r_{k'}}, \gamma_{R_{k'}}))$, type(B) action. Another topological operation is the subtraction $((\Gamma(\gamma_{r_k}, \gamma_{R_k}) \ominus (\Gamma(\gamma_{r_{k'}}, \gamma_{R_{k'}}))$, type(C) action. The characteristics of the interaction causing intra-action is that it retains the geometry of a hyperplane homeomorphism to type(A), type(B), and type(C) spheroids $(\mathbb{H}(\xi, \Delta) \xrightarrow{S}{}^{3} (A, B, C))$, which are stretched or elongated spheroids due to the nature of interactions causing intra-actions.

Axiom 9. Persistency of action: the transitional actions are continuous actions, (axiom 7), with respect to work. There is no gap in the transition from one action type to another. Topologically this states that differential functions representing the transitional process with respect to work done within the zones of radii (r), and (R), (γ_r, γ_R), are all non-zero differential functions. Therefore, the persistency of action guarantees the existence of differential manifolds. Geometrically, the persistency of action is represented as differential manifolds (plates) constituted by differential tensors considered as fibre bundles indicating the direction of the gravitational force of the work within the 0zones of various radii, (r), and (R). The gravitational force of the work ,(γ_r, γ_R) is defined as the weight of different action types, (γ_r, γ_R), $(\gamma_r, \gamma_{r'})$, $(\gamma_r, \gamma_{R'})$, $(\gamma_{r'}, \gamma_R)$, $(\gamma_{r'}, \gamma_{R'})$ in constituting the causal action tensors.

This axiom refers to the existence of differential or tangent planes. Given any set of zones of work, (γ_r, γ_R), of radii (r,R), and given a syntax hyperplane $(\mathbb{H}(\xi, \Delta))$, there exists a homeomorphism $(\mathbb{H}(\xi, \Delta) \xrightarrow{S^3} \Gamma(\gamma_r, \gamma_R))$. (S^3) is an spheroid formulated as $(S^3 = \frac{I^2}{a^2} + \frac{I^2}{b^2} + \frac{I^2}{c^2} = \diagdown^2)$, where $(\diagdown = \{(r, r'), (r, R), (r', R), (r', R'), (r, R')\})$. The differential of the spheroid at any point $(\diagdown)$ is given in an algebraic form as $((S^3)' = \frac{2}{a^2} \cdot \frac{\partial(S^3)}{\partial I_r} + \frac{2}{b^2} \cdot \frac{\partial(S^3)}{\partial v_r} + \frac{2}{c^2} \cdot \frac{\partial(S^3)}{\partial F_r})$. The matrix form of the differential of the spheroid is given in a matrix format as :

$$
\begin{array}{cc}
 & \begin{array}{ccc} I & v & F \end{array} \\
\begin{array}{c} I \\ v \\ F \end{array} & \begin{pmatrix} (\frac{2}{a_r^2})_{1,1} & (\frac{2}{a_r^2})_{1,2} & (\frac{2}{a_r^2})_{1,3} \\ (\frac{2}{b_r^2})_{2,1} & (\frac{2}{b_r^2})_{2,2} & (\frac{2}{b_r^2})_{2,3} \\ (\frac{2}{c_r^2})_{3,1} & (\frac{2}{c_r^2})_{3,2} & (\frac{2}{c_r^2})_{3,3} \end{pmatrix}
\end{array}
\otimes \begin{pmatrix} \frac{\partial(S^3)}{\partial I_r} \\ \frac{\partial(S^3)}{\partial v_r} \\ \frac{\partial(S^3)}{\partial v_r} \end{pmatrix}
$$

$$
= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}
$$

Let the trace of the matrix be given as $(tr^r = (\frac{2}{a_r^2})_{1,1} + (\frac{2}{b_r^2})_{2,2} + (\frac{2}{c_r^2})_{3,3})$. Similar formulations apply to $((r, r'), (r', R), (R', R))$.

The diagonal action tensor of size $((n+m) \times (n+m))$ for the differential plane of the spheroid is given as:

$$
\begin{array}{cc}
 & \begin{array}{cccccc} 1 & \cdots & n & 1 & \cdots & m \end{array} \\
\begin{array}{c} 1 \\ \vdots \\ n \\ 1 \\ \vdots \\ m \end{array} &
\begin{pmatrix}
(tr^r)_{1,1} & \cdots & 0 & (tr^{r'})_{1,1} & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & (tr^r)_{n,n} & 0 & \cdots & (tr^{r'})_{n,m} \\
(tr^{R'})_{1,1} & \cdots & 0 & (tr^R)_{1,1} & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & (tr^{R'})_{m,n} & 0 & \cdots & (tr^R)_{m,m}
\end{pmatrix}
\end{array}
$$

A collection of diagonal action tensors constitute a differential plane. Figure 21, presents a differential plane, and its relevance to the causal cubiod. In Figure 21, the spheroid differential plane cuts the causal cubiod at one of its' action axis depending on the type of action used, where different types of actions are, $(\Gamma(\gamma_r, \gamma_R), \Gamma(\gamma_r, \gamma_{R'}), \Gamma(\gamma_{r'}, \gamma_R), \Gamma(\gamma_{r'}, \gamma_{R'}), \Gamma(\gamma_r, \gamma_{r'}))$.

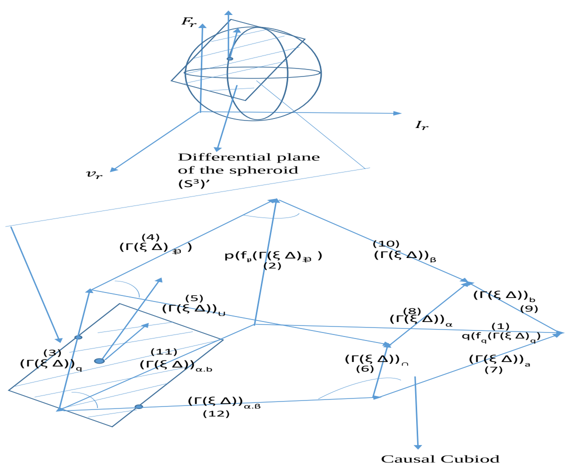


Figure 21. Graphical representation of the persistency of action

Axiom 10. Force of action: this is a gravitational force caused by the following actions; transitional action, (axiom 7), inter-action causing intra-action, (axiom 8), persistency of action, (axiom 9), and the dialectic equilibrium, (axiom 5). Topologically, it can be represented by a tensor function of the algebraic representation of the (4) axioms, namely axioms, (7-9), and (5). Geometrically, it is demonstrated as a torsion, that causes distortion in the causal manifold. Torsion is a twist in the causal manifold causing curved surfaces. The force of action does not cause discontinuity. Discontinuity is caused when causality is outside of the causal space. Discontinuity is considered as a special case to be discussed later.

The hyperplane $(\mathbb{H}(\xi, \Delta))$ is at an angular position. Let's assume that without the effect of axiom (5), the syntax hyperplane is at equilibrium, thus $(\mathbb{H}(\xi, \Delta))$ is in a horizontal position. Then with every variation of the transformation, (evolution) operator, (Δ), and the intermediate intention operator, $(\mathbb{I}_C)$, the position of the causal syntax hyperplane changes. The hyperplane takes various positions represented by angle $((\varpi)_{i,j}; i = 1, \cdots, n; j = 1, \cdots, m)$, and $(0 \leq \varpi \leq 360)$. Therefore, each angle represents a transformation of the basic syntax that provokes variation in action $(\Gamma(\gamma_{r_k}, \gamma_{R_k}); k = 1, 2)$. A graphical example is given in Figure 22.

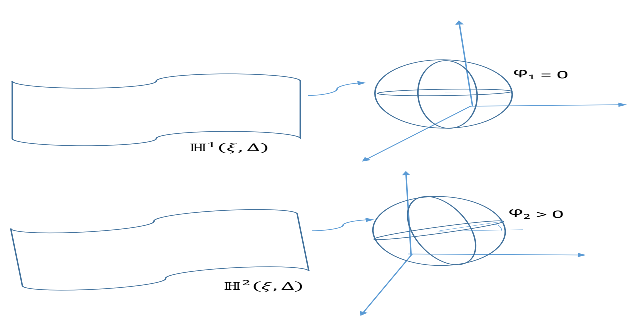


Figure 22. angle variations of spheroids

In Figure 22, the first hyperplane is in a horizontal position, $(\mathbb{H}^1(\xi, \Delta))$, and it's mapping to the three dimensional spheroid, (in the Figure, an sphere is given as a simplified version of an spheroid). $(\varpi_1 = 0)$, the angle with the horizontal line is zero degrees. $(\mathbb{H}^2(\xi, \Delta))$ is the second hyperplane with a tilted position represented by the positive angle, $(\varpi_2 > 0)$ of the sphere in the 3D mapping of the tilted hyperplane.

Axiom 11. Dialectic of Power: this axiom relates to the force of action as a torsion. The dialectic of power is related to the evolution of torsion. As a tensor entity it posses direction and rotation. Torsion evolves by taking varied directions and different rotations. Thus dialectic of power is topologically represented by fibre bundles of the regions of a causal manifold where torsion is applied.

This axiom, dialectic of Power, or (torsion) refers to rotation with an inflection point. Torsion causes deviation in action. Deviation in action is when action changes direction, for example, action that is represented as $(\Gamma(\gamma_r, \gamma_R))$ to action that is represented as $(\Gamma(\gamma_{r'}, \gamma_{R'}))$. This change in action is the rotation, and the inflection of action. The torsion is the inflection point where the direction of action changes. Described as such, torsion becomes an operator tensor that acts on action tensor to enable change in the direction of action when it is required. Rotation-torsion is demonstrated in Figure 23.

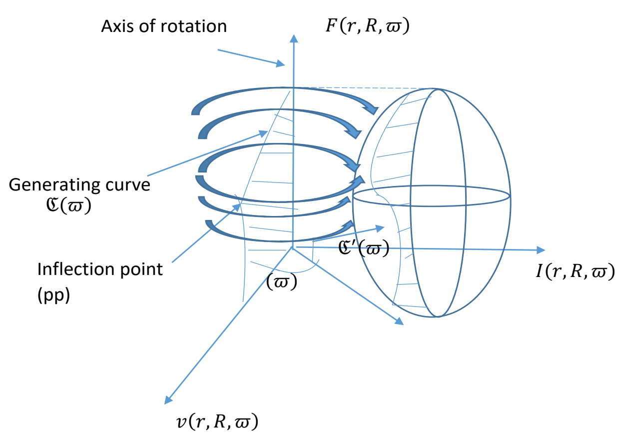


Figure 23. Representation of Torsion

In Figure 23, it is assumed that the spheroid is twisted by rotation, in (2) different directions around the vertical axis (F), and the plane (vF), and the angle of rotation (ϖ). Rotation can occur around any of the axis (v), and (F), here only the vertical twist is considered. The curve $(\mathfrak{C})$ is any section of the spheroid, that is twisted. The twisting of the curve $(\mathfrak{C})$, includes an inflection point (pp). The inflection point is identified by the change in the direction of the differential vectors. The curve is identified through parametrization of the axes (I,v,F) with respect to angle (ϖ). Thus the curve ($\mathfrak{C}$)) becomes ($\mathfrak{C}(\varpi)$), where ($0 \leq \varpi \leq 2\pi$) is the angle of rotation. Torsion applied to action is a tensor operator that acts on the causal action tensor. The first step in constructing a torsion tensor is to build the curve ($\mathfrak{C}(\varpi)$). The purpose is to show how rotation or twist in action can be induced. Twist in action is when a set of actions can be reversed into an opposite set of actions that are performed in different zones of radii, different from the previous ones. The curve ($\mathfrak{C}(\varpi)$) can be considered as a cross section of an spheroid. This cross section curvature is parameterized ($\mathfrak{C}(\varpi) = (\Gamma(\gamma_{\bar{r}}).cos(\varpi), (\Gamma(\gamma_{\bar{r}}).sin(\varpi), (\Gamma(\gamma_{\bar{r}}).(\varpi)))$. Thus the axes are given as ($I_{\bar{r}} = \Gamma(\gamma_{\bar{r}}).cos(\varpi)$), ($v_{\bar{r}} = (\Gamma(\gamma_{\bar{r}}).sin(\varpi)$, and ($F_{\bar{r}} = \Gamma(\gamma_{\bar{r}}).(\varpi))$), where ($\bar{r} = \{(\gamma_r, \gamma_{r'}; (\gamma_r, \gamma_R); (\gamma_r, \gamma_{R'}); (\gamma_{r'}, \gamma_{R'}); (\gamma_{r'}, \gamma_R)$) represents different combinations of zones with various radii of action. Given the parameterizd basis, the spheroid can be re-written in terms of parameterized bases as: ($\frac{(I_{\bar{r}})^2}{a^2} + \frac{(v_{\bar{r}})^2}{b^2} + \frac{(F_{\bar{r}})^2}{c^2} = \bar{r}^2$), where ($\bar{r}$) represents any of the different radii, (r,r',R',R)). The matrix representation of the parameterized cut off section including the curvature ($\mathfrak{C}(\varpi)$) spheroid is given as:

$$\begin{array}{c c} & \begin{array}{ccc} I & v & F \end{array} \\ \begin{array}{c} I \\ v \\ F \end{array} & \begin{pmatrix} (\frac{1}{a_{\bar{r}}^2})_{1,1} & 0 & 0 \\ 0 & (\frac{1}{b_{\bar{r}}^2})_{2,2} & 0 \\ 0 & 0 & (\frac{1}{c_{\bar{r}}^2})_{3,3} \end{pmatrix} \end{array} \otimes \begin{pmatrix} I_{\bar{r}}^2 \\ v_{\bar{r}}^2 \\ F_{\bar{r}}^2 \end{pmatrix}$$

$$= \begin{pmatrix} \bar{r}_1^2 \\ \bar{r}_2^2 \\ \bar{r}_3^2 \end{pmatrix}$$

The matrix form of the differential of the cut off section of the spheroid with the curve ($\mathfrak{C}(\varpi)$), representing differential vectors ($\frac{\partial(I_{\bar{r}})^2)}{\partial(\varpi)}$); ($\frac{\partial(v_{\bar{r}})^2)}{\partial(\varpi)}$); and ($\frac{\partial(F_{\bar{r}})^2)}{\partial(\varpi}$). The differential matrix form denoted as the (B) matrix is given as:

$$B = \begin{array}{c c} & \begin{array}{ccc} I & v & F \end{array} \\ \begin{array}{c} I \\ v \\ F \end{array} & \begin{pmatrix} (\frac{(2)\Gamma(\bar{r})\dot{s}in(\varpi)}{a_{\bar{r}}^2})_{1,1} & 0 & 0 \\ 0 & (\frac{(2)\Gamma(\bar{r})\dot{c}os(\varpi)}{b_r^2})_{2,2} & 0 \\ 0 & 0 & (\frac{(2)\Gamma(\bar{r})\dot{c}os(\varpi)}{c_{\bar{r}}^2})_{3,3} \end{pmatrix} \end{array} \otimes \begin{pmatrix} \frac{\partial I_{\bar{r}}}{\partial t} \\ \frac{\partial v_{\bar{r}}}{\partial t} \\ \frac{\partial F_{\bar{r}}}{\partial t} \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

The determinant of the (B) matrix is given as ($\det(B) = \frac{(2)\Gamma(\bar{r})\dot{c}os(\varpi)}{c)^2}((\frac{(2)\Gamma(\bar{r})\dot{s}in(\varpi)}{a_r^2})_{1,1} \times (\frac{(2)\Gamma(\bar{r})\dot{c}os(\varpi)}{b_r^2} b_r^2)_{2,2})$). The determinant of the differential matrix (B), ($\det(B)$) indicates the direction of the differential vectors on the parameterized curve ($\mathfrak{C}(\varpi)$). Therefore, if the determinant is positive, ($\det(B) > 0$) the differential vector is moving in the positive direction, (clockwise) as is shown in Figure 23, and if the determinant is negative, ($\det(B) < 0$, then the differential vector is moving in the negative direction, (counter clockwise) as is shown in Figure 23. The torsion tensor operator,($\mathfrak{B}$), is a diagonal block matrix of size $((n{+}m)\times(n{+}m))$. As the other operators, this one is made of (4) blocks. Block one matrix is a diagonal matrix of size $(n\times n)$ with the diagonal entries equal to the trace of the differential of matrix (B). The trace of the differential for block one is $(tr_{\mathfrak{B}}(r) = (\frac{(2)\Gamma(r)\dot{s}in(\varpi)}{a_r^2})_{1,1} + (\frac{(2)\Gamma(r)\dot{c}os(\varpi)}{b_r^2})_{2,2} + (\frac{(2)\Gamma(r)\dot{c}os(\varpi)}{c_r^2})_{3,3})$. The trace of the first block relates to action within a zone of radius (r). The second block is a diagonal matrix of size $(n\times m)$, with diagonal entries equal to trace of the differential matrix (B) for action within zones of radius (r'). The trace is given as, $(tr_{\mathfrak{B}}(r') = (\frac{(2)\Gamma(r')\dot{s}in(\varpi)}{a_{r'}^2})_{1,1} + (\frac{(2)\Gamma(r')\dot{c}os(\varpi)}{b_{r'}^2})_{2,2} + (\frac{(2)\Gamma(r')\dot{c}os(\varpi)}{c_{r'}^2})_{3,3})$. The third block is a diagonal matrix of size $(m\times n)$, with diagonal entries relating to action within zones of radius (R'). The trace is given as: $(tr_{\mathfrak{B}}(R') = (\frac{(2)\Gamma(R')\dot{s}in(\varpi)}{a_{R'}^2})_{1,1} + (\frac{(2)\Gamma(R')\dot{c}os(\varpi)}{b_{R'}^2})_{2,2} + (\frac{(2)\Gamma(R')\dot{c}os(\varpi)}{c_{R'}^2})_{3,3})$. The fourth matrix block is a diagonal matrix of size $(m\times m)$. The diagonal entries are the trace matrix (B) relating to action within zones of radius (R), $(tr_{\mathfrak{B}}(R) = (\frac{(2)\Gamma(R)\dot{s}in(\varpi)}{a_R^2})_{1,1} + (\frac{(2)\Gamma(R)\dot{c}os(\varpi)}{b_R^2})_{2,2} + (\frac{(2)\Gamma(R)\dot{c}os(\varpi)}{c_R^2})_{3,3})$. The torsion tensor operator is given as:

$$\mathfrak{B} = \begin{array}{c} \\ 1 \\ \vdots \\ n \\ 1 \\ \vdots \\ m \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & \cdots & n & 1 & \cdots & m \end{array} \\ \begin{pmatrix} (tr_{\mathfrak{B}}^{r,\varpi})_{1,1} & \cdots & 0 & (tr_{\mathfrak{B}}^{r',\varpi})_{1,1} & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & (tr_{\mathfrak{B}}^{r,\varpi})_{n,n} & 0 & \cdots & (tr_{\mathfrak{B}}^{r',\varpi})_{n,m} \\ (tr_{\mathfrak{B}}^{R',\varpi})_{1,1} & \cdots & 0 & (tr_{\mathfrak{B}}^{R,\varpi})_{1,1} & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & (tr_{\mathfrak{B}}^{R',\varpi})_{m,n} & 0 & \cdots & (tr_{\mathfrak{B}}^{R,\varpi})_{m,m} \end{pmatrix} \end{array}$$

Axiom 12. Gravitational Centrality, (GC): this axiom relates to how action can gravitate towards specific centers. In the context of the (DCTA) economics, these centers of gravitation are thought centers represented outwardly by a transformed syntax, and are mapped onto spatial centers of physical environments. Geometrically, the centers of gravitation of action are the regions of the causal manifold, where there exists dips or curvatures and where the fibre bundles are at maximum.

Action gravitates towards certain centers. A center is defined as a point on the parameterized curve $(\mathfrak{C}(\varpi))$ where the differential vectors are at optimal points of a causal syntax manifold. This translates to the points with differentials equal to zero. The differential vector becomes the tangent vector. Thus these centers are the points where action is optimal. It is assumed that intention verbs, that indicate action gravitate towards zones where the causal syntax hyperplane, $(\mathbb{H}(\xi,\Delta))$ has curvature, and where tangent vectors exist, in other words, the maximum points of curvature. Consequently, it is posited that syntax privileges optimal actions and its' combinations. Axiom 12 is similar to axiom 11, with the exception, that the differential vectors are tangents to the centers of gravitation. Gravitational Centrality, is demonstrated in Figure 24.

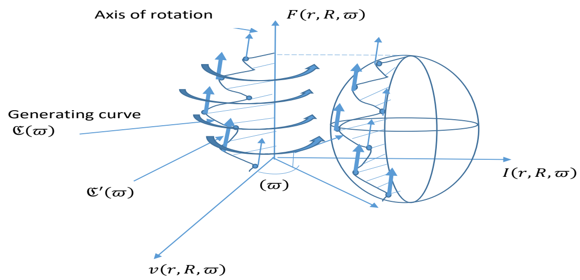


Figure 24. Representation of Gravitational Centrality

The Gravitational Centrality tensor operator denoted by $(\mathfrak{G})$ is similar to the torsion tensor operator of axiom 11, with diagonal entries of each matrix block equal to the derivatives with respect to angle (ϖ) equal to zero. The trace in general becomes $(tr_{\mathfrak{G}}(\bar{r}) = (\frac{(2)\Gamma(\bar{r})\dot{s}in(\varpi_{max})}{a_{\bar{r}}^2})_{1,1} + (\frac{(2)\Gamma(\bar{r})\dot{c}os(\varpi_{max}}{b_{\bar{r}}^2})_{2,2} + (\frac{(2)\Gamma(\bar{r})\dot{c}os(\varpi_{max}}{c_{\bar{r}}^2})_{3,3})$, where (ϖ_{max}) is the angle for which the differential is equal to zero. Thus for the sine function $(sin(\varpi))$, the derivative is zero at $(\varpi_{max} = 0, \pi, 2\pi)$, and for the cosine function the derivative is zero at $(\varpi_{max} = \frac{\pi}{2}, \frac{3\pi}{2})$. The Gravitational Centrality tensor operator, $(\mathfrak{G})$ is given as:

$$\mathfrak{G} = \begin{array}{c c} & \begin{array}{cccccc} 1 & \cdots & n & 1 & \cdots & m \end{array} \\ \begin{array}{c} 1 \\ \vdots \\ n \\ 1 \\ \vdots \\ m \end{array} & \begin{pmatrix} (tr_{\mathfrak{G}}^{r,\varpi_{max}})_{1,1} & \cdots & 0 & (tr_{\mathfrak{G}}^{r',\varpi_{max}})_{1,1} & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & (tr_{\mathfrak{G}}^{r,\varpi_{max}})_{n,n} & 0 & \cdots & (tr_{\mathfrak{G}}^{r',\varpi_{max}})_{n,m} \\ (tr_{\mathfrak{G}}^{R',\varpi_{max}})_{1,1} & \cdots & 0 & (tr_{\mathfrak{G}}^{R,\varpi_{max}})_{1,1} & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & (tr_{\mathfrak{G}}^{R',\varpi_{max}})_{m,n} & 0 & \cdots & (tr_{\mathfrak{G}}^{R,\varpi_{max}})_{m,m} \end{pmatrix} \end{array}$$

Axiom 13. Pressure Zones, (PZ): pressure zones are similar to the Gravitational Centers, (GC). The difference is that topologically, (PZ)s are where torsion tensors are modified by tensor operators, in order to show the tendency of a causal action manifolds towards the (GC)s. Geometrically, (PZ)s are the curved regions indicating the tendency of a causal manifold in having a (GC). In the context of of the (DCTA) economics, Pressure Zones are expressed as a thought process through a syntax operator, (transformation operator), (Δ), and intention intermediate operator,$(\mathbb{I}_C)$, applied to a syntax, and are mapped spatially onto action through intention operator, $(\mathbb{I})$, thus as such demonstrate the prevalence of a certain type of a distinct action such as, (micro, macro, and other types of transitional actions) within a physical environment.

The axioms 11, and 12 are embedded in axiom 13. This axiom refers to a probabilistic tensor operator. The probability calculated as the probability of the occurrence of inflection points and or optimal points. The probability of the occurrence of an inflection point can be demonstrated by different types of actions given as

$(\Gamma(\gamma_{\bar{r}}, \gamma_{\bar{R}}))$,where $(\bar{r} = r, r')$, and $(\bar{R} = R, R')$. These actions are signified by the radii of zones, $(\bar{r})$, and $(\bar{R})$. Let's denote a different set of actions by $(\Gamma(\gamma_{\bar{r}^*}, \gamma_{\bar{R}^*}))$, where $(\bar{r}^* \neq \bar{r})$, and $(\bar{R}^* \neq \bar{R})$, and let's use these action types to describe the probabilities. Let the inflection point occur at the point with circular zones of radii $(\bar{r}_0, \bar{R}_0)$ and $(\bar{r}_0^*, \bar{R}_0^*)$ of action given a specific group of syntax, $(s_A^i \in G_A; i = n_0; n_0 < n)$; and $(s_B^j \in G_B; j = m_0; m_0 < m)$.

The probability that inflection occurs given a syntax group (n_0, m_0) relates to the probability of change from action type $(\Gamma(\gamma_{\bar{r}}, \gamma_{\bar{R}}))$, to action type $(\Gamma(\gamma_{\bar{r}^*}, \gamma_{\bar{R}^*}))$ is given as $(p_{\mathfrak{B}}(\Gamma(\gamma_{\bar{r}_0^*}, \gamma_{\bar{R}_0^*}) \mid \Gamma(\gamma_{\bar{r}_0}, \gamma_{\bar{R}_0})))$, the probability of the change of action $(\Gamma(\gamma_{\bar{r}^*}, \gamma_{\bar{R}^*}))$ given action type $((\Gamma(\gamma_{\bar{r}_0}, \gamma_{\bar{R}_0}))$ is the conditional probability formulated as $(p_{\mathfrak{B}}(\Gamma(\gamma_{\bar{r}_0^*}, \gamma_{\bar{R}_0^*}) \mid \Gamma(\gamma_{\bar{r}_0}, \gamma_{\bar{R}_0})) = \frac{p(\Gamma(\gamma_{\bar{r}_0^*}, \gamma_{\bar{R}_0^*}) \cap (\Gamma(\gamma_{\bar{r}_0}, \gamma_{\bar{R}_0})}{\sum_i^n \sum_j^m p(\Gamma(\gamma_{\bar{r}}, \gamma_{\bar{R}})})$, given $(\Gamma(\gamma_{\bar{r}_0^*}, \gamma_{\bar{R}_0^*}) \in (s_A^{n_0}, s_B^{m_0}))$. The probability of the occurrence of inflection at $(\Gamma(\gamma_{\bar{r}_0^*}, \gamma_{\bar{R}_0^*})$ given any action type, $(\Gamma(\gamma_{\bar{r}}, \gamma_{\bar{R}}))$ is calculated as $(p_{\mathfrak{B}}(\Gamma(\gamma_{\bar{r}_0^*}, \gamma_{\bar{R}_0^*}) \mid \Gamma(\gamma_{\bar{r}}, \gamma_{\bar{R}})) = \frac{p(\Gamma(\gamma_{\bar{r}_0}, \gamma_{\bar{R}_0})\dot{p}(\Gamma(\gamma_{\bar{r}_0^*}, \gamma_{\bar{R}_0^*}))}{\sum_i^n \sum_j^m p(\Gamma(\gamma_{\bar{r}_i}, \gamma_{\bar{R}_j})}) = \frac{(tr(\bar{r}_0 + tr(\bar{R}_0)\dot{(}tr(\bar{r}_0^* + tr(\bar{R}_0^*)}{\sum_i^n \sum_j^m (tr(\bar{r}_i + tr(\bar{R}_j)}))$.

The optimal points are those points on the parameterized curve $(\mathfrak{C}(\varpi))$ where the differential vector is equal to zero demonstrated in axiom 12. For the sake of demonstration, let the optimal point be at (r_{opt}, R_{opt}) such that given $(s_A^i \in G_A; i = n_{opt}; n_{opt} < n)$; and $(s_B^j \in G_B; j = m_{opt}; m_{opt} < m)$, the probability of the occurrence of an optimal point given action $(\Gamma(\gamma_r, \gamma_R))$ is given as the conditional probability of the occurrence of an optimal action at a point, $((\Gamma(\gamma_{r_{opt}}, \gamma_{R_{opt}}))$, given action type $(\Gamma(\gamma_r, \gamma_R)))$, $(p_{\mathfrak{C}}((\Gamma(\gamma_{r_{opt}}, \gamma_{R_{opt}}) \mid \Gamma(\gamma_r, \gamma_R))))$. This conditional probability is calculated as: $(p_{\mathfrak{C}}(\Gamma(\gamma_{r_{opt}}, \gamma_{R_{opt}}) \mid \Gamma(\gamma_r, \gamma_R)) = \frac{p(\Gamma(\gamma_{r_{opt}}, \gamma_{R_{opt}})\dot{p}(\Gamma(\gamma_r, \gamma_R)}{\sum_i^n \sum_j^m \mathfrak{p}(\Gamma(\gamma_{r_i}, \gamma_{R_j}}} = \frac{(tr(r_{opt}) + tr(R_{opt})\dot{(}tr_{n_{opt}}(r) + tr_{m_{opt}}(R)}{\sum_i^n \sum_j^m (tr(r_i + tr(R_j)})$. The trace of the optimal point is calculated in axiom 12.

The probability of the occurrence of an inflection point, in tensor operator, $(\mathfrak{B})$ is denoted as $(\mathfrak{B}_{(\Gamma(\gamma_{r_0^*}, \gamma_{R_0^*})})$, and is formulated as $(p_{\mathfrak{B}}(\Gamma(\gamma_{r_0^*}, \gamma_{R_0^*}) \mid \Gamma(\gamma_r, \gamma_R)))$ can be developed for all sub-sets of $(s_A^i \in G_A)$.and $(s_B^j \in G_B)$ in order to construct a diagonal block matrix of size $((n+m) \times (n+m))$ as is shown below:

$$
\mathfrak{B}_{(\Gamma(\gamma_{r_0^*}, \gamma_{R_0^*})} = \begin{array}{c} \\ 1 \\ \vdots \\ n \\ 1 \\ \vdots \\ m \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & \cdots & n & 1 & \cdots & m \end{array} \\ \begin{pmatrix} (p_{\mathfrak{B}}(r))_{1,1} & \cdots & 0 & (p_{\mathfrak{B}}(r^*))_{1,1} & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & (p_{\mathfrak{B}}(r))_{n,n} & 0 & \cdots & (p_{\mathfrak{B}}(r^*))_{n,m} \\ (p_{\mathfrak{B}}(R^*))_{1,1} & \cdots & 0 & (p_{\mathfrak{B}}(R))_{1,1} & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & (p_{\mathfrak{B}}(R^*))_{m,n} & 0 & \cdots & (p_{\mathfrak{B}}(R))_{m,m} \end{pmatrix} \end{array}
$$

The probability of the occurrence of an inflection point is calculated for each block given action within zones of different radii. Similarly, the probability of the occurrence of an optimal point, in (GC) tensor operator, ($\mathfrak{G}$), is denoted as ($\mathfrak{G}_{(\Gamma(\gamma_{r_{opt}},\gamma_{R_{opt}})}$), and is formulated as ($p_{\mathfrak{G}}((\Gamma(\gamma_{r_{opt}},\gamma_{R_{opt}}) \mid \Gamma(\gamma_r,\gamma_R))))$ is developed for all sub-sets of (L_A), and (L_B). The outcome is a diagonal block tensor of size $((n+m)\times(n+m))$ as is shown below:

$$\mathfrak{G}_{r_{opt},\gamma_{R_{opt}})} = \begin{array}{c} \\ 1 \\ \vdots \\ n \\ 1 \\ \vdots \\ m \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & \cdots & n & 1 & \cdots & m \end{array} \\ \begin{pmatrix} (p_{\mathfrak{G}}(r))_{1,1} & \cdots & 0 & (p_{\mathfrak{G}}(r_{opt}))_{1,1} & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & (p_{\mathfrak{G}}(r))_{n,n} & 0 & \cdots & (p_{\mathfrak{G}}(r_{opt}))_{n,m} \\ (p_{\mathfrak{G}}(R_{opt}))_{1,1} & \cdots & 0 & (p_{\mathfrak{G}}(R))_{1,1} & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & (p_{\mathfrak{G}}(R_{opt}))_{m,n} & 0 & \cdots & (p_{\mathfrak{G}}(R))_{m,m} \end{pmatrix} \end{array}$$

These tensors are used to locate the special points, (inflection, and optimal) included in the syntax , (ξ,Δ). As operators, they act on the intermediate intention operator, ($\mathbb{I}_C$), multiplied by the intention operator, ($\mathbb{I}$), it is mapped onto the (3) dimensional space of the spheroids that gives the dominant action types, $(\Gamma(\gamma_{\bar{r}},\gamma_{\bar{R}}); \bar{r} = r, r'; \bar{R} = R, R')$, that corresponds to one of the action axes of the causal cuboid.

Axiom 14. Physical environment: refers to maneuvers, and movements dealing with natural obstacles, small and large man-made obstacles, machinery, equipment, etc. Topologically, physical environment represents a special causal set of the (Noun) group, and has a fixed category. This category is used as a syntax operator to modify any initial syntax, (ξ) to (ξ,Δ). The transformation operator, (Δ) includes sub groups of the (Noun) group. Modified syntax causes transitional action. Modification in the type of action is expressed in terms of work done within a small and large radii. This axiom is implicit in axioms 11 - 13.

5 DCTA Economics re-constructed demand,consumer-producer surplus, supply, equilibrium, and supply-demand growth

Movement is a geometrical manifestation of a syntax. Syntax is a physical manifestation of a thought process. Based on the premise of the (DCTA), a syntax manifests itself geometrically as a complex set of movements. The variations in movements is due to the dialectic nature of a syntax. These movements are the causal aspect of the (DCTA) demand. This means that the space, the magnitude, and the variations of the required movements determines the quantity demanded, and subsequently the acceptable price per unit consumed for an individual. Given this proposition, we can introduce the (DCTA) demand, and its modules consumer surplus. The (DCTA) supply is also given, along with modules of equilibrium, and growth.

In this section the connection between work leading to action, and the fundamentals of the micro economics, namely, demand, (D), and supply, (S), plus the side products, consumer surplus, (χ) equilibrium,(E) and growth, (G) as it relates to the evolution of demand and supply are discussed. The tool used to set up the kernel or the germ of the space relating action, (Γ), to quantities demanded, (q_D), and to quantities supplied,(q_S), and price per quantity demanded (p_D), and price per quantity supplied (p_S) is the causal cuboid, denoted as (V_c). The particularity of (V_c) is that it contains a total of (12) axes, out of which(10), $(1, \cdots, 10)$ axes are different combinations of actions, $(\Gamma(\gamma_{\bar{r}}, \gamma_{\bar{R}}))$, where $(\bar{r} = (r', r))$, and $(\bar{r} = (R', R))$ are various radii considered for circular zones where work is performed, $(\gamma_{\bar{r}})$, and $(\gamma_{\bar{R}})$. As a reminder, (r) is the radius of a small circular zone representing micro actions, (γ_r) produced by a sub-set (L_A). $(\gamma_{r'})$ represents a circular zone with a small radius (r') within a zone of large radius, (R) representing micro actions produced by a sub-set (L_B). (γ_R) represents a circular zone with a large radius (R) representing macro actions produced by a sub-set (L_B). $(\gamma_{R'})$ represents a circular zone with a large radius (R') representing macro actions within a small zone of radius (r), produced by a sub-set (L_A). In general action is represented by circular zones of various radii, $(\bar{r})$, and $(\bar{R})$. Axis (1), is the axis that represents quantity demanded for a product that is a function of action, thus demand is constructed based on action, $(D = D(\Gamma(\bar{r}, \bar{R}))$. The quantity demanded depends on the action performed, $(q_D = q_{D(\Gamma(\bar{r},\bar{R})})$. The same axis also denotes the supply side, when supply is a function of action, $(S = S(\Gamma(\bar{r}, \bar{R}))$. The quantity supplied is an element depending on action denoted as $(q_S = q_{S(\Gamma(\bar{r},\bar{R})})$. Axis (2), is the axis that represents price per quantity demanded. Price is a function of action, $(p_D = p_{D(\Gamma(\bar{r},\bar{R})})$. Pricing from the supplier (producer) is a function of suppliers' action, and is denoted by is $(p_S = p_{S(\Gamma(\bar{r},\bar{R})})$.

To connect action to demand and supply, given a causal cuboid the following procedure is adopted. The first step is to map a causal conceptual tensor manifold, $(\mathbb{H}(\xi, \Delta))$ in $((n + m))$ dimension $(\Re^{(n+m)})$, that contains all $((n + m))$ sub-sets, $(s^i_A \in L_A)$, and $(s^j_B \in L_B)$ in a syntax, onto causal action tensors in the 3 dimensional spheroid $(S^3 \subset \Re^3)$, $(\mathbb{H}(\xi, \Delta) \xrightarrow{S^3} \Gamma(\gamma_{\bar{r}}))$. Depending on which type of action is considered, which is reflected in which zones are the causal action tensors, one or several of the action axes are used. Each point on the tensor axis of a causal cuboid corresponds to one of the (10) action types. Figure 25, shows this correspondence. In Figure 25, the syntax hyperplane is mapped onto the (3) dimensional causal action spheroids. The causal action spheroids, correspond to one of the axes of the causal cuboid, (V_c), and intersects the corresponding axis at a fixed point, here denoted as point (a_1) on axis (11). In Figure 25, an example of such points is given in order to convey the general mapping from the (3D) spheroids, to the axes of the (V_c). Point (a_1),corresponds to the quantity demanded, (q^1_D), the quantity supplied, (q^1_S), and the price per quantity demanded, (p^1_D), and the price per quantity supplied, (p^1_S). Depending on the type of the spheroid , other action tensors axes can be chosen.

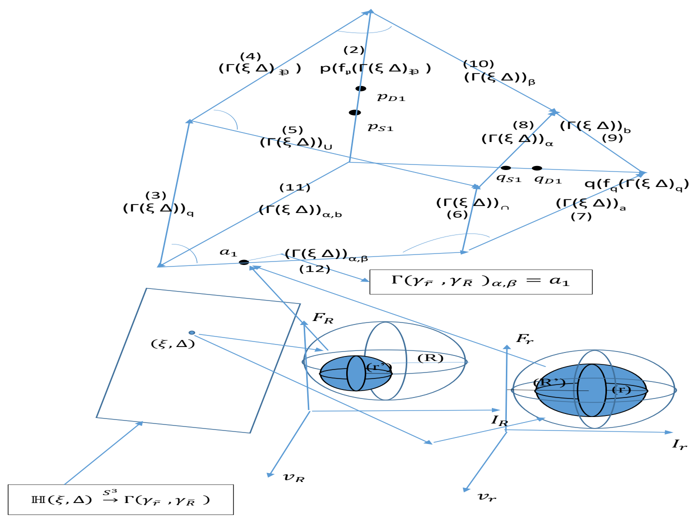


Figure 25. Correspondence between causal action spheroids, (S^3) and the causal cuboid, (V_c)

The connection between action, (Γ),(q_D), the quantity supplied, (q_S), and the price per quantity demanded, (p_D), and the price per quantity supplied, (p_S) is demonstrated in Figure 26.

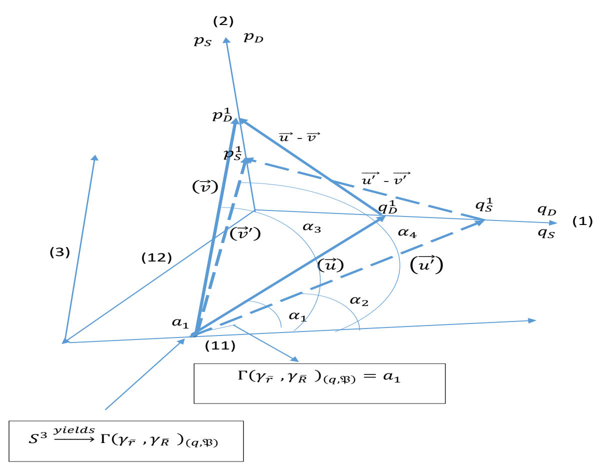


Figure 26. Demonstration of how action, (Γ), determines (q_D),q_S), and (p_D),(p_S)

In Figure 26, axis (1), is action relating to demand. Given the syntax (ξ, Δ) in the causal action tensor hyperplane, ($\mathbb{H}(\xi, \Delta)$), is mapped onto a spheroid, that is projected to the tensor axis (11) at the point (a_1). (a_1) is mapped onto the axis (1) or the axis that corresponds to actions that leads to quantity consumed,and quantity produced, denoted by $((\Gamma(\gamma_{\bar{r}}, \gamma_{\bar{R}}))_{q,\mathfrak{p}})$. Axis (1) is denoted as both the axis of the quantity of consumption, (q_D), and the axis of the quantity of production, (q_S). From the point (a_1) a vector $(\vec{u})$ is drawn towards axis (1) or the quantity axis, (this axis represents either the quantity demanded or the quantity supplied). Vector $(\vec{u})$ connects the point (a_1) to the point (q_D^1), the quantity demanded on axis 1. Vector $(\vec{u'})$ connects the point (a_1) to the point (q_S^1), the quantity supplied on axis 1. The points (q_D^1), and (q_S^1), are either stated in the syntax, or depend on the angle between the vector $(\vec{u})$ and axis (1), and the vector, $(\vec{u'})$and axis (1).

Let's denote the angle between the vector $(\vec{u})$, and axis (11) by $(\alpha_1; 0 < \alpha_1 < \frac{\pi}{2})$, where angle, (α_1) is between zero and $(\frac{\pi}{2})$. Let's denote the angle between the vector $(\vec{u'})$, and axis (11) by $(\alpha_2; 0 < \alpha_2 < \frac{\pi}{2})$, where angle, (α_2) is between zero and $(\frac{\pi}{2})$. Given the magnitude of the angle, (α_1), there exists a range of quantities demanded that can be identified, as $((q_D^1, q_D^2, \cdots, q_D^l)$, where (l) represents the limit of the quantities demanded based on the vector $(\vec{u})$'s angle with axis (11). Similarly, given the magnitude of the angle, (α_2), there exists a range of quantities supplied that can be identified, as $((q_S^1, q_S^2, \cdots, q_S^{ll})$, where (ll) represents the limit of the quantities supplied based on the vector $(\vec{u'})$'s angle with axis (11). Vector $(\vec{v})$ connects axis (11), at point (a_1) to the point, (p_D^1), price per quantity demanded on axis (2). Axis (2) represents both the price per unit demanded,(p_D), and the price per unit supplied,(p_S). Similarly, Vector $(\vec{v'})$ connects axis (11), at point (a_1) to the point, (p_S^1), price per quantity demanded on axis (2). This point, (p_D^1) is identified in the syntax. If the price is not directly stated in the syntax, a range of prices that can be identified through the angle (α_3), $(\alpha_3; 0 < \alpha_3 < \frac{\pi}{2})$, which gives price ranges,$((p_D^1, p_D^2, \cdots, p_D^l)$, corresponding to each quantity demanded identified. This point, (p_S^1) is identified in the syntax. If the price is not directly stated in the syntax, a range of prices that can be identified through the angle (α_4), $(\alpha_4; 0 < \alpha_3 < \frac{\pi}{2})$, which gives price ranges,$((p_S^1, p_S^2, \cdots, p_S^{ll})$, corresponding to each quantity supplied is identified.

Vector $(\vec{u} - \vec{v})$ connects the quantity demanded (q_D^1) to the corresponding price (p_D^1). In the case of several quantities demanded and their corresponding prices, similar vectors $((\vec{u_1} - \vec{v_1}), (\vec{u_2} - \vec{v_2}), \cdots, (\vec{u_l} - \vec{v_l}))$ connect the corresponding pairs of points. Vector $(\vec{u'} - \vec{v'})$ connects the quantity supplied (q_S^1) to the corresponding price (p_S^1). In the case of several quantities supplied and their corresponding prices, similar vectors $((\vec{u'_1} - \vec{v'_1}), (\vec{u'_2} - \vec{v'_2}), \cdots, (\vec{u'_l} - \vec{v'_l}))$ connect the corresponding pairs of points. The triangle $(a_1 q_D^1 p_D^1)$ is the demand triangle. Each demand triangle is in $(\Re^{(n+m)})$ dimension. Thus $((a_1 q_D^1 p_D^1) \in \Re^{(n+m)})$. The same applies to the case of multiple demand triangles. The causal supply $(S = S(\Gamma(\bar{r}, \bar{R}))$ is a triangle constructed by three vectors $(\vec{u'})$, $(\vec{v'})$. The triangle $(a_1 q_S^1 p_S^1)$ is the supply triangle. Each supply triangle is in $(\Re^{(n+m)})$ dimension. Thus $((a_1 q_S^1 p_S^1) \in \Re^{(n+m)})$. The same applies to the case of multiple supply triangles. The pairs (q_D^1, p_D^1), and (q_S^1, p_S^1) are pairs of points identified by the mapping of the syntax onto action $(\mathbb{H}(\xi, \Delta) \xrightarrow{S^3} \Gamma(\gamma_{\bar{r}}, \gamma_{\bar{R}}))$. The vectors that make both the causal demand , and the causal supply triangles are tensors. This is due to the various interpretations of the syntax, and its' multifaceted possible transformations. Therefore, the causal demand and supply triangles are hyperplanes of dimension ($\Re^{(n+m)}$). The possible modifications of the syntax due to the application of the transformation operator (Δ) leads to the range of quantities demanded, and the range of their corresponding prices, and the range of quantities supplied, and the range of their corresponding prices. This is mainly due to the differentials of the action tensor using axioms (11 -13). The optimal quantities demanded and supplied are found using axioms (11-13), as well as the inflection point(s). The characteristic of the optimal point is that it is superior to any quantity demanded and supplied. Axioms (11-13) are used to find the inflection points of the quantity demanded and the quantity supplied. The inflection point is the point where both quantities change due to the rotational movement of the action spheroid around one of the three action axes that leads to change in consumption or production. Both the causal demand triangle, and the causal supply triangle are given in Figure 26.

Consumer surplus is found given the optimal quantity demanded, (q_D^{max}), and the optimal price per unit quantity demanded, (p_D^{max}). The optimal quantity demanded, (q_D^{max}), is obtained, given point, (a_1) by rotating the angle (α_1) of the vector ($\vec{u}$) close to ($\frac{\pi}{2}$), close to the axis (11). The optimal price per unit demanded, (p_D^{max}) is obtained in a similar way by rotating the angle (α_2) of the vector ($\vec{v}$) close to ($\frac{\pi}{2}$), farthest from the axis (11). The change of angle is done through an interpretation of the syntax through operators using axioms (11-13). The condition is that the quantity demanded is less than the maximum quantity demanded, ($q_D^{max} > q_D$), and the price per quantity demanded, is less than the maximum price per quantity demanded, ($p_D^{max} > p_D$). The optimal demand hyper triangle (($a_1 q_S^{max} p_S^{max}$)), is obtained. Given the hyper triangle of Figures 25,and 26, (($a_1 q_S^1 p_S^1$)), the consumer surplus is the region equal to the difference between the area of the hyper triangle ($\Delta(a_1 q_S^{max} p_S^{max})$), and the hyper triangle ($\Delta(a_1 q_S^1 p_S^1)$), ($\Delta(a_1 q_S^{max} p_S^{max} - \Delta(a_1 q_S^1 p_S^1)$. The shaded area, the (ribbon highlighted) is the consumer surplus, shown in Figure 26+. The producer surplus is the inverse of consumer surplus. The producer surplus which is related to supply hyperplane triangle, increases as the consumer acceptable price, (p_D), gets closer to the maximum acceptable price, (p_D^{max}). The assumption is that the maximum consumer acceptable price per unit of consumption is equal to the producer's optimal price per unit of production, (p_S^{max}). In general the producer surplus is the strip equal to the difference between the area of the hyperplane triangle ($\Delta(a_1 q_S p_S)$), and the desirable hyperplane triangle ($\Delta(a_1 q_S p_S)$), ($\Delta(a_1 q_S^{max} p_S^{max}) - \Delta(a_1 q_S p_S)$). As the consumer surplus ribbon gets narrower, the producer surplus ribbon gets wider. This is demonstrated in Figure 26+, and Figure 27.

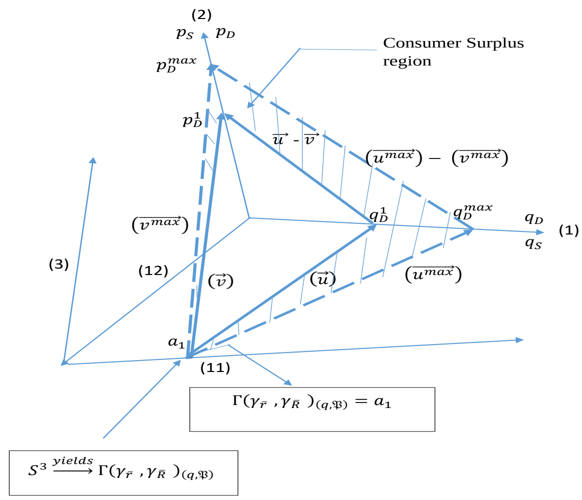


Figure 26+. Consumer Surplus

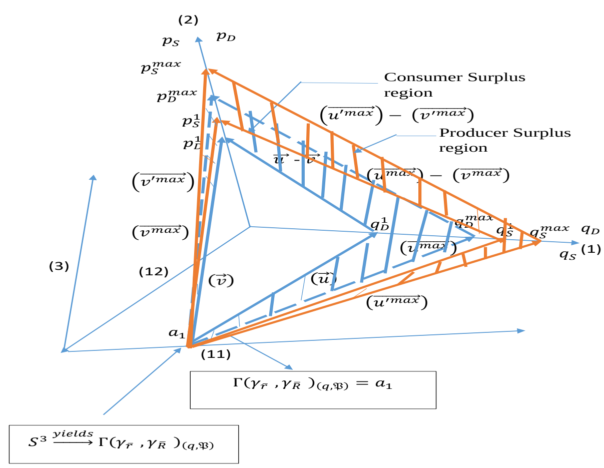


Figure 27. Consumer Surplus compared to producer surplus

The bases that constitute the causal cuboid are not orthogonal to each other. The axes on the causal cuboid (V_c) are skewed with respect to each other. All the 12 axes stand at different angles from each other. The orthogonal axes are independent axes. The causal axes of the (V_c) are functions of action, $(\Gamma(\gamma_r, \gamma_R))$. There exists covarience causal tensor operators that determine the position of the causal axes with respect to each other. The co-variance causal tensor operators are diagonal block tensor of size $((n+m)+(n+m))$. There are 4 block matrices. Block (1) is an $(n \times n)$

matrix with entries given as $(cov(\Gamma(\gamma_r, \gamma_{R'}))$. Block (2) is an $(n \times m)$ matrix with entries given as $(cov(\Gamma(\gamma_{r'}, \gamma_R))$. Block (3) is an $(m \times n)$ matrix with entries given as $(cov(\Gamma(\gamma_{R'}, \gamma_{r'}))$. Block (4) is an $(m \times m)$ matrix with entries given as $(cov(\Gamma(\gamma_R, \gamma_r))$.

$(cov(\Gamma(\gamma_r, \gamma_{R'}), \Gamma(\gamma_{r'}, \gamma_R)))$ is given as

$$\begin{array}{cc}
 & \begin{matrix} 1 & \cdots & n & 1 & \cdots & m \end{matrix} \\
\begin{matrix} 1 \\ \vdots \\ n \\ 1 \\ \vdots \\ m \end{matrix} &
\begin{pmatrix}
(cov(\Gamma(\gamma_r, \gamma_{R'}))_{1,1} & \cdots & 0 & (cov(\Gamma(\gamma_{r'}, \gamma_R))_{1,1} & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & (cov(\Gamma(\gamma_r, \gamma_{R'}))_{n,n} & 0 & \cdots & (cov(\Gamma(\gamma_{r'}, \gamma_R))_{n,m} \\
(cov(\Gamma(\gamma_{R'}, \gamma_{r'}))_{1,1} & \cdots & 0 & (cov(\Gamma(\gamma_R, \gamma_r))_{1,1} & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & (cov(\Gamma(\gamma_{R'}, \gamma_{r'}))_{m,n} & 0 & \cdots & (cov(\Gamma(\gamma_R, \gamma_r))_{m,m}
\end{pmatrix}
\end{array}$$

where $((cov(\Gamma(\gamma_r, \gamma_{R'})) = \frac{\partial(\Gamma(\gamma_r, \gamma_{R'})}{\partial \gamma_r} \mathbin{\centerdot} \frac{\partial(\Gamma(\gamma_r, \gamma_{R'})}{\partial \gamma_{R'}})$ are the diagonal covariences of the first block. $((cov(\Gamma(\gamma_{r'}, \gamma_R)) = \frac{\partial(\Gamma(\gamma_{r'}, \gamma_R}{\partial \gamma_{r'}} \mathbin{\centerdot} \frac{\partial(\Gamma(\gamma_{r'}, \gamma_R}{\partial \gamma_R})$ are the diagonal covariance entries of the 2nd block. $(cov(\Gamma(\gamma_{R'}, \gamma_{r'}) = \frac{\partial(\Gamma(\gamma_{R'}, \gamma_{r'}}{\partial \gamma_{R'}} \mathbin{\centerdot} \frac{\partial(\Gamma(\gamma_{R'}, \gamma_{r'}}{\partial \gamma_{r'}})$ are the diagonal entries of the 3rd block of the covarience causal tensor operator. $((cov(\Gamma(\gamma_R, \gamma_r)) = \frac{\partial(\Gamma(\gamma_R, \gamma_r)}{\partial \gamma_R} \mathbin{\centerdot} \frac{\partial(\Gamma(\gamma_R, \gamma_r}{\partial \gamma_{R'}})$ is the diagonal entries of the 4th block of the covariance tensor. The covariance operators can be constructed between any two axes of the causal cuboid. For the axes of the causal cuboid that do not have a point in common, their position with each other is determined by their covariance with their adjacent axis. This means a 2 stage covariance calculation. Stage (1), is the calculation of the covariance with an adjacent tensor. Stage (2) the second covariance with the adjacent to the other axis is calculated. Thus the covariance between two non-adjacent axes is the sum of the the (2) covariances with the axis that they both share a point in common.

The position of the demand triangle in $(\Re^{(n+m)})$ dimension changes as the angles between the axes of the causal cuboid change. In general, the angle, $(\measuredangle \Gamma_l, \Gamma_{l'})$, where (Γ_l) represents axis (l), and $(\Gamma_{l'})$ represents any axis other than (l), $(lneql')$, denoted by (α_l) of the two adjacent axes with a point in common is calculated as the inner product of the two tensors, (Γ_l), and $(\Gamma_{l'})$. The angle between any two adjacent axes,$(l, l+1)$ can be calculated using the inner product equation of two vectors, or tensors, $(\Gamma_l, \Gamma_{l'})$, where (Γ_l), and $(\Gamma_{l'})$ represent two adjacent tensors, then $(\varpi_{l,l'} = \frac{|\Gamma_l \centerdot \Gamma_{l'}|}{|\Gamma_l| \centerdot |\Gamma_{l'}|} \times \pi)$, where $(|\Gamma_l \centerdot \Gamma_{l'}|)$ is the absolute value, or the length of the inner product of the two tensors, and $(|\Gamma_l|)$ which s the length of the tensor (Γ_l), and $(|\Gamma_{l'}|)$, which is the length of the adjacent tensor.

The case of mixed actions axis, for example axis 11, which represents a fractional mixed of actions relating to demand and supply, formulated as $(a(\Gamma(\gamma_{\bar{r}}, \gamma_{\bar{R}}))_q + b(\Gamma(\gamma_{\bar{r}}, \gamma_{\bar{R}}))_{\mathfrak{p}})$, the relationship with quantity demanded and price per unit demanded , and supply, and price per unit of production are shown in Figure 28. In this case, both a fraction of the actions of the individual, and a fraction of the actions of the producer are taken into account simultaneously. In other words, both the individual, and the producer are aware of some fraction of each others actions. The individual adapts his quantity demanded based on his expectation of the quantity supplied, which translates to either the quantity demanded is assumed greater than the quantity supplied, $(q_D > q_S)$ or the quantity demanded is assumed less than the quantity supplied, $(q_D < q_S)$. The same assumptions apply to price. Therefore, the price per quantity demanded is either greater than the price desired by the producer,$(p_D > p_S)$ or the price per quantity demanded is less than the price desired by the producer, $p_D < p_S)$.

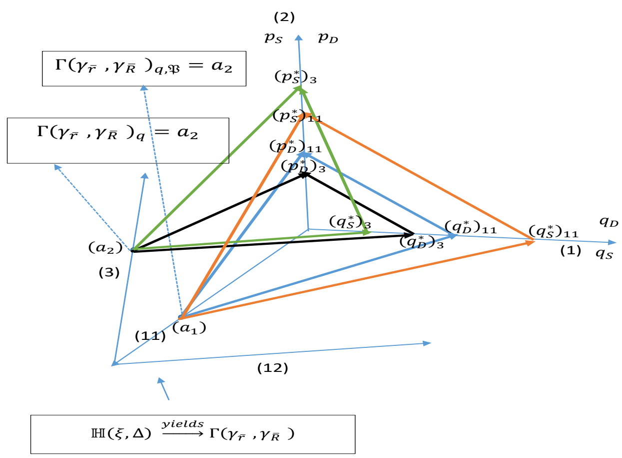


Figure 28. Demonstration of how mixed actions determine quantities demanded, and quantities supplied

In the example shown in Figure 28, triangles are constructed from point (a_1)on a mixed action axis (11), and a point on the consumption related actions axis (3). The purpose of this exercise is to show the difference between demand and supply constructed based on a single type action, axis (3), and the mixed type action, axis (11). The blue color hyper triangle $((a_1 q *_D p*_D))$ is the causal demand hyper triangle constructed from the mixed action types, and the red hyperplane triangle,$((a_1 q *_S p*_S)_{red})$, is the hyperplane supply triangle. The two triangles, the green hyperplane triangle,$((a_2 q *_D p*_D))$, is the demand hyperplane triangle, and the black hyperplane triangle, $((a_2 q *_S p*_S))$ is the supply hyperplane triangle. The red and the blue triangles reflect that both the consumer , and the producer have some knowledge of each others' types of actions, while green, and black triangles reflect that the actions of consumers and producers are independent of each other. The independence of consumer and producer actions is taken into account when calculating the points on axis (1), the demand axis, and the price axis, (2). Both are the possible causal supply hyper triangles perceived as the producers' response to the individual demand hyper triangle. Both of the supply hyper triangles are valid producer response. In this case both supply hyper triangles are accepted as supply. If the producer did not consider the actions of the individual, then the producer's supply hyper triangle would be the black triangle.

Equilibrium,(E) and growth, (G) as it relates to the evolution of demand and supply are calculated as follows. Equilibrium, (E) in the context of the DCTA economics equilibrium has (2) levels. An internal Equilibrium, (IE), and an External Equilibrium, (XE). (IE) is reached when the hyperplane demand triangle intersects the hyperplane supply triangle as is shown in Figure 29. As is shown in Figure 29, there are (2) cases in the (IE). Case (1), is when $(q_D < q_S)$, and $(p_D < p_S)$. In this case no equilibrium exists, even though consumer and producer surpluses are shown. Case (2) is when $(q_D > q_S)$, and $(p_D > p_S)$. There exists a point of intersection, (A) is either the right side intersection of the (2) hyperplane triangles, or it is on the left side of the intersection of the (2) hyperplane triangles. The location of (IE) depends on the angle (ϖ), which is the angle between the axis of action with the other axes of action, including the quantity (q), and the price (p) axes. Point (A) in the context of tensor demand and supply hyperplane triangles becomes a spherical p-gons shown in Figure 29. This is due to the dimension s of the hyperplane triangles, and due to the hyperplane triangles being constructed by tensors, which makes the demand and supply, tensor base hyper triangles. All points on the spherical p-gons are (IE) points, and each of the p-gons, or faces which are either triangles, or rectangles represent the relationship between the (IE) points.

Theorem 5.1. *Given an spherical p-gons, let the number of vertices be denoted by (N_0), and the p-gons be denoted by $(\{p\})$, and the number of p-gons attached to a vertex,be denoted by $(\{q\})$, then (N_0), and $\begin{Bmatrix} p \\ q \end{Bmatrix}$ on any spherical p-gons, (SP) depend on the choice of the action axes of the (V_c), the causal cuboid.*

Proof. The (IE) spherical p-gons must have the following characteristics: 1) The (SP) is equilateral, meaning that all arcs are of equal length. 2) The (SP) is equiangular, then the sphere is a regular sphere. 3) Each vertex of the set of vertices, (N_0), contains p-arcs, taken as p-tensors. 4) The (SP) is of type $\begin{Bmatrix} p \\ q \end{Bmatrix}$. 5) Given the (4) characteristics above, the (SP) becomes a convex spherical p-gons. 6) The existence of an Internal Equilibrium, (IE), depends on the intersection of the demand, and the supply hyper triangles as is shown in Figure 29, given as point (A). 7) Point (A) is the center of the convex (SP). 8) Mixed actions axes where fractions, and probabilities are included are the cause of congruent transformation of the (SP). In the case where a single action type axis is used, the (IE) point (A), becomes the center of a regular sphere rather than a congruent (SP). Mixed actions axes correspond to a congruent (SP) with $\begin{Bmatrix} p \\ q \end{Bmatrix}$, normally equal to $\begin{Bmatrix} 4 \\ 3 \end{Bmatrix}$, and $\begin{Bmatrix} 4 \\ 4 \end{Bmatrix}$. This means that every vertex on the (SP) shares (4) triangles, a demand, and supply triangles pair, either on the same side or on the opposite sides of a congruent (SP), and equally each vertex shares (4) rectangles pairs either on the same side or on the opposite sides of a congruent (SP). The rectangular shapes occur when more than one mixed actions axes, or a combination of mixed actions, and single action axes are used from a (V_c). Mixed actions, and combination mixed, and single action axes cause a convex (SP) to become congruent and allows congruent transformations. Congruent transformation includes translation, rotation, and reflection. It is given that the (IE) point, (A), is the center of a congruent (SP), found from the original or base demand, and supply hyper triangles intersection. The (IE) point (A) is located with reference to the homeomorphic mapping from $(\mathbb{R}^{(n+m)})$ to $(\mathbb{R}^3)$, thus to a given trihedron (simply a triangle) by its oblique Cartesian coordinates (x,y,z). Any vertex sharing $\left\{ \begin{array}{c} 4 \\ 3 \end{array} \right\}$ denoted by $(A' \in N_0)$ shares the same oblique coordinates (x,y,z). Since the first, (base) trihedron is fixed due to the mapping process, then the vertex sharing $\left\{ \begin{array}{c} 4 \\ 3 \end{array} \right\}$ is fixed. Thus every (IE) point (A), determines a unique vertex,$(A' \in N_0)$. This is a congruent transformation. The second process is the rotation around the (IE) point, (A). Since the (IE) point (A) is fixed, then rotation is equivalent to reflection of (A), if rotation is in one direction, otherwise it is a simple rotation. Any vertex in the set (N_0),sharing array, $\left\{ \begin{array}{c} 4 \\ 3 \end{array} \right\}$, is considered as a reflection of the original point (A). Thus it is considered as an equivalent of the (IE) point, (A). Thus, based on the properties assumed due to the invariance of the (IE) point (A), and the transformation, rotation, and reflection properties, the number of vertices (N_0) and thus the number of $\left\{ \begin{array}{c} p \\ q \end{array} \right\}$ being either $\left\{ \begin{array}{c} 4 \\ 3 \end{array} \right\}$ or $\left\{ \begin{array}{c} 4 \\ 4 \end{array} \right\}$ depends on the possible transformations and reflections possible, which become functions of fractions (a,b), and or probabilities (α, β) used to formulate mixed actions. □

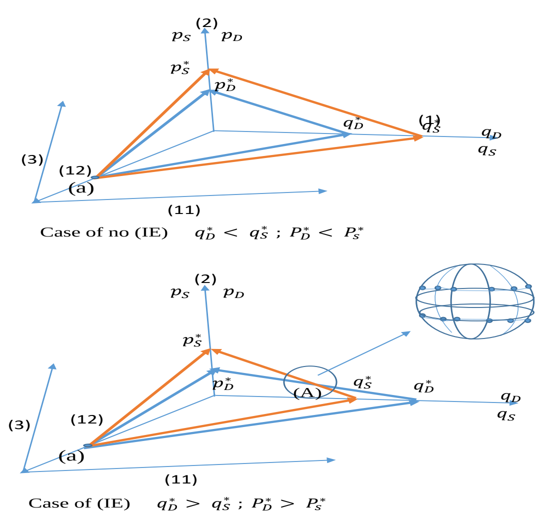


Figure 29. Internal Equilibrium (IE)

(XE) is reached if the evolution of syntax implies the possibility of additional types of actions that can be attached to the base (V_c) do not cause a change in the areas of the demand, and supply triangles and thus do not change the number of (IE) points or the number of faces(p) of the equilibrium spherical p-gons.An example is shown in Figure 30. Point (a) is the point on axis (11). A demand and a supply hyper triangle is constructed from this point. Due to the evolution of syntax, a new action axis can be added to the base causal cuboid, (V_c). Given any point (a') on the new action axis, demand and supply hyper triangles are constructed. If the (2) constructed hyper triangles have an (XE), then the (XE) by definition is a spherical p-gons, with different number of nodes, and faces,(p)s. In general, (XE) should occur, when after every evolution of a syntax, the tensor demand and supply hyper triangles possess a common region, namely an (XE). The (XE) is in the form of an spherical p-gons. The existence of an (XE) does not require that the new spherical p-gons to be equivalent to the previous one. This is shown in Figure 30.

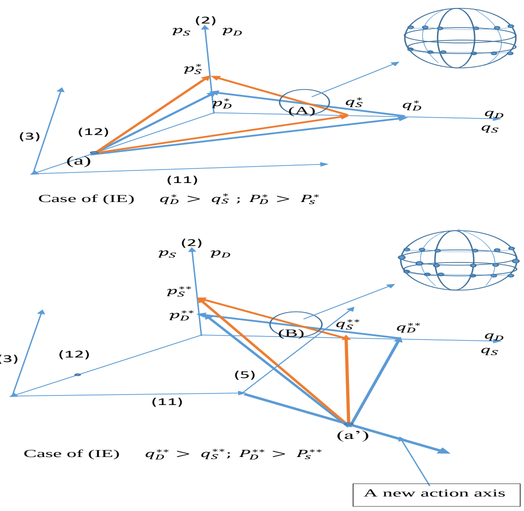


Figure 30. External Equilibrium (XE)

Micro economic growth occurs when both demand and supply tensor hyper triangles multiply. Multiplication happens when more in number or in their complication of actions become causality for demand and supply due to the evolution of a syntax either through transformation (Δ), or evolution through the repeat of the occurrence of any of the (14) DCTA axioms. The new causal action tensors constitute a new causal cuboid (V_c'), thus a new quantity (q) axis, and a new price (p) axis calculated with the same functional forms. If the area of the new demand and supply tensor hyper triangles are greater that the previous ones, the micro economic growth occurs. Thus growth is detected as a comparison between the areas of (2) consecutive demand and supply tensor hyper triangles. This is shown in Figure 31.

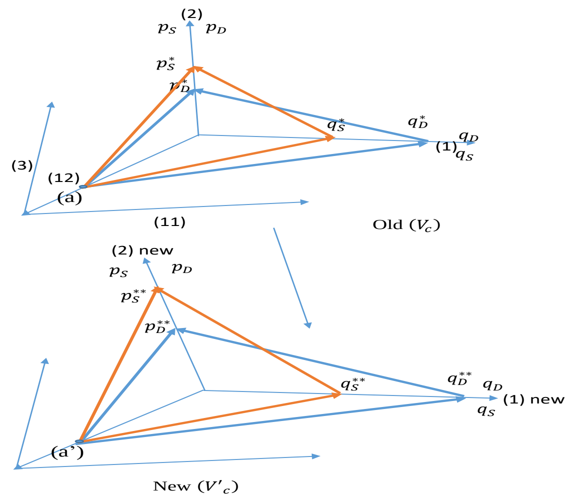


Figure 31. micro economic growth

6 Conclusion

In this paper a new economic theory, (DCTA economics) is proposed that is based on syntax mapped onto action. (DCTA) is constructed based on (14) axioms. These (14) axioms provide a tool that converts syntax to action. Action leads to either consumption or production. Thus action is constructed using syntax and its' possible interpretations and transformations. Thus syntax, and its' transformations considered as dialectic are the cause of action. Action is the causality of consumption and production. The space that relates action to consumption and production is an $(\Re^n \otimes \Re^m)$ tensor causal space where both syntax (ξ, Δ), and its' induced action $(\Gamma(\gamma_{\bar{r}}, \gamma_{\bar{R}}))$, where $(\bar{r} = r, r')$, and $(\bar{R} = R, R')$ are tensors of size $((n+m) \times (n+m))$. A mapping from an $(\mathbb{R}^{(n+m)})$ space to $(\mathbb{R}^3)$space allow for the quantitative measurement of action. This mapping allows the formulation of the demand, and the supply as a function of action. The causal relationship between action and demand and supply is constructed as a cuboid called a causal cuboid (V_c). Each side of the causal cuboid (V_c) is an axis tensor of dimension $((n+m) \times (n+m))$. There are 12 sides or axes in (V_c) of which two are the quantity demanded and supplied axis, and the price per quantity demanded, and price per quantity supplied. Given the syntax space, as a cause of actions, demand and supply are formulated and given in a geometrical format. Other related entities, consumer-producer surplus, supply, equilibrium, and supply-demand growth are re-defined in the context of DCTA economics.

References

[1] J. H. Aguilar, A. A. Buckareff, *Causing Human Actions: New perspectives on the Causal Theory of Action*, Massachusetts Institute of Technology, Cambridge, MA. 2010.

[2] D. H. Mellor, *The facts of Causation*, Routledge publisher,London, UK 1995.

[3] P. M. Schuhl, *L'oeuvre de Platon*, Hachette publisher,Paris, FR 1962.

[4] J. Locke, *An Essay Concerning Human Understanding*, Penguin Books,London, UK 1997.

[5] Hegel, *Science de la Logique, La Doctrine du Concept*, Edition Kimé,Paris,FR 1962.

[6] M. Heidegger, *L'oeuvre de Platon*, Editions Gallimard,Paris, FR 1942.

[7] Kant, *Critique de la raison pure*, Editions Flammarion,Paris, FR 1942.

[8] L. Wittgenstein, *Tractatus Logico-Philosophicus*, Penguin Classics,London, UK 1926.

[9] R. Scruton, *Modern Philosophy*, Penguin Books,London, UK 1994.

[10] M. Heidegger, *Recherches Philosophiques*, Editions Gallimard,Paris, FR 2001.

[11] L. Wittgenstein, *Tractatus Logico-Philosophicus*, Penguin Classics,London, UK 1926.

[12] N. Chomsky, *Structures Syntaxiques* , Editions Seuil, Mouton and Company, La Haye, The Netherlands 1957.

[13] N. Chomsky, *Aspects de la théorie syntaxiqu*, Editions Seuil, Paris, FR 1971.

[14] J. Lyons, *Linguistiques générale syntaxique*, Editions Larousse, Paris, FR 1970.

[15] E. Nowak, *Philosophy of Language: the basics*, Routledge,New York, NY 2025.

[16] A. Abeillé, *Les Nouvelles Syntaxes*, Editions Armand Coline, Paris, FR 1993.

[17] A. Culioli, *Pour une Linguistique de l'Enonciatiion*, Editions Ophrys, Paris, FR 1990.

[18] S. Pinker, *Rationality*, Penguin Books,London, UK 2022.

[19] G. H. Gadamer, *La Verité et méthode: Les grandes ligne d'une herméneutique philosophique*, Editions Seuil, Paris, FR 1996.

[20] W. Child, *Causality, Interpretation, and the Mind*, Clarendon Press ,Oxford, UK 1994.

[21] D. Davidson, *Causal Relations*, Journal of Philosophy ,No. 64, 1980.

[22] D. H. Mellor, *The facts of Causation*, Routledge Press, London, UK 1995.

[23] C. Moya, *The Philosophy of Action*, Polity Press, Oxford, UK 1990.

[24] T. Nadelhoffer, *Praise, side effects, and intentional action*, Journal of Theoretical and Philosophical psychology , No. 24 , 2004.

[25] A. Noë, *Action in Perception*, MIT Press,Cambridge, MA 2004.

[26] E. Pacherie, *The Phenomenology of Action: A conceptual framework*, Journal of Cognition, No. 107 , 2008.

[27] D. Parfit, *Reasons and Motivation*, Proceedings of the Aristotelian Society, Vol. 71, 1997.

[28] C. Peacocke, *Deviant causal chains*, Midwest Studies in Philosophy, Vol. 4, 1979.

[29] P. Pietroski, *Causing Actions*, Oxford University Press,Oxford, UK 2002.

[30] J. Raz, *Introduction. In Practical Reasoning*, Oxford University Press,Oxford, UK 1978.

[31] M. Rowlands, *Body Language*, MIT Press,Cambridge, MA 2006.

[32] D. Ruben, *Action and its Explanation*, Oxford University Press,Oxford, UK 2003.

[33] C. Sandis, *New Essays on the Explanation of Action*, Basingstoke, Palgrave, Macmillan Press,NY, 2009.

[34] J. Schaffer, *Contrastive causation*, Philosophical Review, Vol. 114, 2005.

[35] G. F. Schueler, *Reasons and Purposes: Human Rationality and the Teleological Explanation of Action* , Clarendon Press ,Oxford, UK 2003.

[36] P. Gärdenfors, *The Geometry of Meaning: Semantics based on conceptual spaces*, MIT Press,Cambridge, MA 2017.

[37] M. M. Khoshyaran, *Homology Theory on Causal Random Groups*, JPRM, Vol. 14 No. 1 (2020) 1-22.

[38] M. M. Khoshyaran, *The Causal Theory of Action based utility and consumer surplus* , JPRM, Vol. 18 No. 4 (2021) 1-22.

[39] H. B.Curry, *Foundations of Mathematical Logic* , Dover publication, NY. 1977.

Author's address:

M. M. Khoshyaran
Research and development Unit,
Economics Traffic Clinic -ETC Paris, France.
E-mail: etclinic@wanadoo.fr