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Original Paper

Entanglement Reconsidered

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Abstract

Geometric algebra formalism, along with unambiguous definition of states, opens the door to developing a feasible mathematical structure to replace conventional quantum mechanics. The three-sphere $\mathbb{S}^3$ becomes the playground of the states as torsion kind ones, eliminating abstract Hilbert space vectors. The $\mathbb{S}^3$ points act in measurements on observable as operators.

Keywords: geometric algebra, states, observables, measurements

1. Introduction

Weirdness of all conventional quantum mechanics comes from logical inconsistence of what is meant in basic quantum mechanical definitions and has nothing to do with the phenomena scale and attached artificial complementarity principles.

It will be stated below that theory should speak about proper splitting of **experimental arrangements** into operator (state or wave function in conventional terminology) acting on observable, and operand, measured observable components.

The novel features are:

- Replacing complex numbers by elements of even subalgebra of geometric algebra in three dimensions, that's by elements of the form "scalar plus bivector".

- Operators acting on observable objects are points on the unit three-sphere $\mathbb{S}^3$ defining rotations in a giving plane by given angle. Those points are connected, due to the negative result of the hedgehog theorem on $\mathbb{S}^3$, by parallel (Clifford) translations.

- Evolution of the $\mathbb{S}^3$ operators by Clifford translations is particularly governed by generalization of the Schrodinger equation with unit bivectors in three dimensions instead of formal imaginary unit.

In the suggested formalism complex numbers $x + iy$ are replaced with elements of even subalgebra of G_3 – geometric algebra in three dimensions.

Even subalgebra G_3^+ is subalgebra of elements of the form $M_3 = \alpha + I_S\beta$, where α and β are (real) scalars and I_S is some unit bivector, arbitrary placed in three-dimensional space. Elements of G_3^+ can be depicted as in Figure 1.

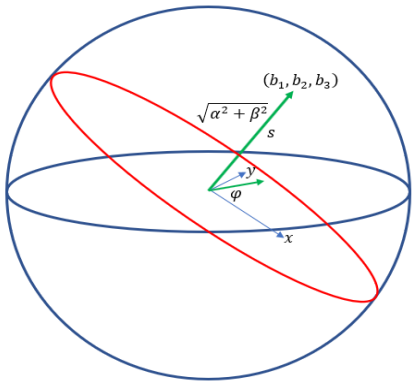

Figure 1. An element of G_3^+

Unit value elements of G_3^+, when $\alpha^2 + \beta^2 = 1$, will be called *g-qubits*. The wave functions (states in the suggested approach) implemented as g-qubits store much more information than quantum mechanical qubits, see Figure 2.

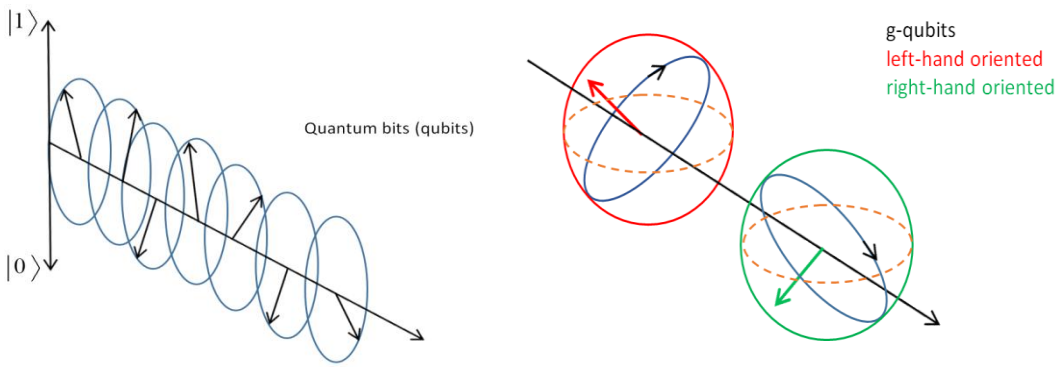

Figure 2. Geomectrically picted qubits and g-qubits

Take right-hand screw oriented basis $\{B_1, B_2, B_3\}$ of unit value bivectors, with the multiplication rules $B_1B_2 = -B_3$, $B_1B_3 = B_2$, $B_2B_3 = -B_1$, $I_3B_1I_3B_2I_3B_3 = I_3$ (or equivalently $B_1B_2B_3 = 1$), where I_3 is oriented unit value volume, pseudoscalar, in three dimensions, see Figure 3.

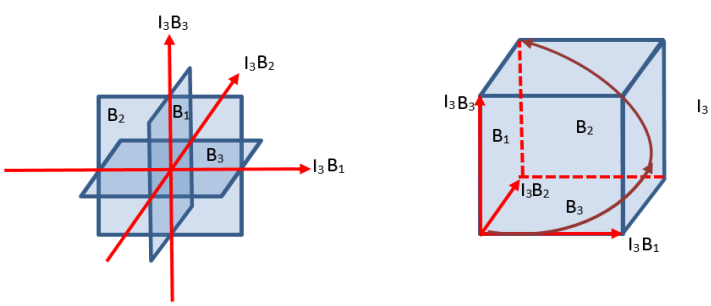

Figure 3. Basis of bivectors, dual vectors and unit value pseudoscalar

The g-qubit state $\alpha + I_S\beta$ can be written then as

$\alpha + \beta(b_1B_1 + b_2B_2 + b_3B_3) \equiv \alpha + \beta_1B_1 + \beta_2B_2 + \beta_3B_3,\ \alpha^2 + \beta_1^2 + \beta_2^2 + \beta_3^2 = 1,$

that gives map onto the unit sphere $\mathbb{S}^3$: $\{\alpha, \beta_1, \beta_2, \beta_3\} \in \mathbb{S}^3$.

Now we can construct exactly three continuous, nowhere-vanishing, and pair-wise orthogonal vector fields, because $\mathbb{S}^3$ is parallelizable maniforld:

$$T_1 = (\alpha + \beta_1B_1 + \beta_2B_2 + \beta_3B_3)B_1 = -\beta_1 + \alpha B_1 - \beta_3B_2 + \beta_2B_3$$

$$T_2 = (\alpha + \beta_1B_1 + \beta_2B_2 + \beta_3B_3)B_2 = -\beta_2 + \beta_3B_1 + \alpha B_2 - \beta_1B_3$$

$$T_3 = (\alpha + \beta_1B_1 + \beta_2B_2 + \beta_3B_3)B_3 = -\beta_3 - \beta_2B_1 + \beta_1B_2 + \alpha B_3$$

This is trivial tangent bundle.

All that guarantees smooth connection between any two g-qubit states as ponts on $\mathbb{S}^3$.

2. Method

Since a state $\alpha + I_S\beta$ parameters satisfy $\alpha^2 + \beta^2 = 1$, we can formally write

$\alpha + I_S\beta = \cos\varphi + I_S \sin\varphi$

Take known expansions with a scalar parameter:

$$\cos\varphi = 1 - \frac{\varphi^2}{2!} + \frac{\varphi^4}{4!} - \cdots$$

$$\sin\varphi = \varphi - \frac{\varphi^3}{3!} + \frac{\varphi}{5!} - \cdots$$

$$e^{\varphi} = 1 + \frac{\varphi}{1!} + \frac{\varphi^2}{2!} + \frac{\varphi^3}{3!} + \cdots$$

Assuming necessary absolute convergence of the series, take unit bivector I_S, multiply by it the sinus series, and add it with the cosine series. Then we get, remembering that $I_S^2 = -1$:

$$e^{I_S\varphi} = \cos\varphi + I_S \sin\varphi$$

Let's have two states, $e^{I_{S_1}\varphi_1}$ and $e^{I_{S_2}\varphi_2}$. Either of them, for example the second one, can be received from the first one:

$$e^{I_{S_2}\varphi_2} = (e^{I_{S_2}\varphi_2}e^{-I_{S_1}\varphi_1})e^{I_{S_1}\varphi_1}$$

The product $e^{I_{S_2}\varphi_2}e^{-I_{S_1}\varphi_1}$ is product of two unit value elements of G_3^+, proof is available, for example, see Sec.1.3 of (Soiguine, 2020):

$$\left|e^{I_{S_2}\varphi_2}e^{-I_{S_1}\varphi_1}\right| = \left|e^{I_{S_2}\varphi_2}\right|\left|e^{-I_{S_1}\varphi_1}\right|$$

Multiplication of an exponent by another exponent is called Clifford translation. Using the term translation follows from the fact that Clifford translation does not change distances between the exponents it acts upon when we identify exponents as points on unit sphere $\mathbb{S}^3$:

$$cos\,\alpha + I_S\, sin\,\alpha = cos\,\alpha + b_1\, sin\,\alpha\, B_1 + b_2\, sin\,\alpha\, B_2 + b_3\, sin\,\alpha\, B_3$$
$$\Leftrightarrow \{cos\,\alpha\,, b_1\, sin\,\alpha\,, b_2\, sin\,\alpha\,, b_3\, sin\,\alpha\}$$

$$(cos\,\alpha)^2 + (b_1\, sin\,\alpha)^2 + (b_2\, sin\,\alpha)^2 + (b_3\, sin\,\alpha)^2 = 1$$

This result follows again from the product property of unit value elements of G_3^+: $|g_1g_2| = |g_1||g_2|$:

$$|e^{I_S\alpha}(g_1 - g_2)| = |e^{I_S\alpha}||g_1 - g_2| = |g_1 - g_2|$$

Explicit form of the connecting Clifford translation is the following one, see (Soiguine, 2020), p.30:

$e^{I_{S_2}\varphi_2}e^{-I_{S_1}\varphi_1} =$

$cos\varphi_2 \, cos\, \varphi_1 + (s_1 \cdot s_2) sin\, \varphi_2 \, sin\, \varphi_1 + I_3 s_2 \, sin\, \varphi_2 \, cos\, \varphi_1 + I_3 s_1 \, cos\, \varphi_2 \, sin\, \varphi_1 + I_3 (s_2 \times s_1) \, sin\, \varphi_2 \, sin\, \varphi_1,$

where s_1 and s_2 are vectors dual to I_{S_1} and I_{S_2}.

The results of all not made measurements of some observable C by any state $e^{I s_2 \varphi_2}$ can be simultaneously received from the result of measurement by $e^{I s_1 \varphi_1}$:

$$e^{-I s_2 \varphi_2} C e^{I s_2 \varphi_2} = e^{-I s_2 \varphi_2} e^{I s_1 \varphi_1} (e^{-I s_1 \varphi_1} C e^{I s_1 \varphi_1}) e^{-I s_1 \varphi_1} e^{I s_2 \varphi_2}$$

This is what entanglement means in the suggested structure.

3. Results

In the Introduction section the trivial tangent bundle was derived. Let's use tangent components for infinitesimal displacements along a big circle on $\mathbb{S}^3$, the circle is assumed without loss of generality to be intersection of the sphere by plane of B_3.

Let corresponding Clifford translation act on some state F: $F \to e^{B_3 \psi} F$. The translational velocity is:

$$\frac{\partial}{\partial \psi} \left(e^{B_3 \psi} F \right) = B_3 e^{B_3 \psi} F$$

It is orthogonal to $e^{B_3 \psi} F$.

Two other tangent components, orthogonal to $e^{B_3 \psi} F$ and $B_3 e^{B_3 \psi} F$, are $B_1 e^{B_3 \psi} F$ and $B_2 e^{B_3 \psi} F$. Their own velocities while moving along Clifford orbit are:

$$\frac{\partial}{\partial \psi} \left(B_1 e^{B_3 \psi} F \right) = B_1 B_3 e^{B_3 \psi} F = B_2 e^{B_3 \psi} F$$

$$\frac{\partial}{\partial \psi} \left(B_2 e^{B_3 \psi} F \right) = B_2 B_3 e^{B_3 \psi} F = -B_1 e^{B_3 \psi} F$$

These two equations explicitly show that the two tangents, orthogonal to Clifford translation velocity, rotate in moving plane $\{B_1 e^{B_3 \psi} F, B_2 e^{B_3 \psi} F\}$ with the same unit value rotational velocity (see Figure 4).

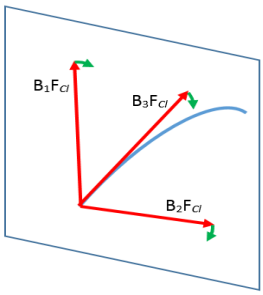

Figure 4. Tangents rotate in their plane with the same, by value, speed as translational velocity

The triple of the translational velocity and two rotational velocities have orientation opposite to the triple of tangents: if the tangents $\{B_1 e^{B_3 \psi} F, B_2 e^{B_3 \psi} F, B_3 e^{B_3 \psi} F\}$ have right screw orientation, the speed triple $\{-B_1 e^{B_3 \psi} F, B_2 e^{B_3 \psi} F, B_3 e^{B_3 \psi} F\}$ is left screw.

If state makes full circle in Clifford translation, both $B_1 e^{B_3 \psi} F$ and $B_2 e^{B_3 \psi} F$ also make full rotation in their own planes by 2π.

Let's take some unit bivector I_{S_i} and make infinitesimal Clifford translation of an arbitrary g-qubit state:

$$\alpha + \beta I_S \to e^{I_{S_i} d\varphi} (\alpha + \beta I_S)$$

Let's move along some arbitrary path on $\mathbb{S}^3$. The sphere is center symmetrical surface, so at any instant value of state on $\mathbb{S}^3$ infinitesimal incrementing of translational velocity angle does not depend in what direction the displacement happens. If we initially approximate the path with infinitesimal pieces of big circle geodesics then the accumulated angle between translational velocity and instant geodesic is obviously equal to the total length of the path (see Figure 5).

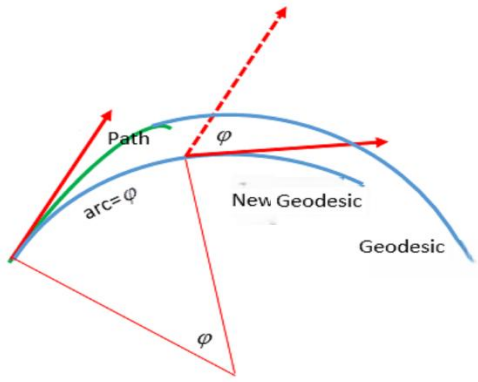

Figure 5. Accumulating of angle while moving along path. Also equals by value to rotation angle of two vectors in plane remaining orthogonal to translational velocity

Infinite composition of infinitesimal Clifford translations along any $\mathbb{S}^3$ path L with varying I_{S_i} gives the final state g-qubit.

Take a sequence of infinitesimal Clifford transformations:

$$e^{I_{S_i}(l_N)\Delta l_N} \dots e^{I_{S_i}(l_2)\Delta l_2} e^{I_{S_i}(l_1)\Delta l_1}(\alpha + \beta I_S)$$

By taking the logarithm, approaching $N \to \infty$ and getting back to exponent we receive the final state:

$$\oint e^{I_{S_i}(l)dl}(\alpha + \beta I_S)$$

This is the G_3^+ kind of holonomy. Measurement of an observable C by this state will be:

$$\overline{(\alpha + \beta I_S)} \oint e^{-I_{S_i}(l)dl} C \oint e^{I_{S_i}(l)dl}(\alpha + \beta I_S)$$

4. Discussion

Further work will be on the Berry phase/Berry connections inside the geometrical algebra structure successfully replacing conventional quantum mechanics formalism. The results already available demonstrate an opportunity of a kind of super-computing that is superior to entangle based standard quantum computing, particularly eliminating all huge, and unresolved, problems of the latter. Upcoming plans include establishing a joint venture for developing and marketing the product. Tough problem is to keep hidden highly valuable intellectual property.

5. Conclusions

It was demonstrated that the geometric algebra formalism along with generalization of complex numbers to geometrically feasible elements of even subalgebra of geometric algebra in three dimensions allows clear and not mysterious explanation of entanglement. This weirdness of entanglement is milestones of all further difficulties in interpretation of conventional quantum mechanics.

References

Soiguine, A. (2020). *The Geometric Algebra Lift of Qubits and Beyond*. LAMBERT Academic Publishing.