World Journal of Business Research
E-ISSN 2770-9078
Vol. 2, No. 1, 2022
Published by STSL Press

Original Paper

Precise Assessment of the World Economic System Stability Using Mandelbrot Set

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Abstract

There are different models that describe behavior of the world economic system. In this work, we use Discrete Dynamic Model (DDM) in order to assess the world economic system stability (sensitivity of its growth to alteration of its parameters). DDM represents an expansion of a function, describing the dependence of the rate of GDP change in the current year on the rate of GDP change in the previous year, in a Taylor series. Its formal expansion up to the polynomial of degree 3 helps to describe qualitatively certain subtle effects of the world GDP dynamics. Coefficients of expansion of the approximating polynomial were determined by the method of least squares using actual values of the rate of the world GDP change. Analysis of characteristics of the approximating polynomials obtained on different time intervals allowed assessing the world economic system stability. We demonstrated earlier that stable states of the world economic system could be characterized by: (1) an attractive fixed point, (2) periodic points or (3) a strange attractor. DDM allows performing a deeper assessment of the world economic system sensitivity to a change in key macro-economic parameters that influence the coefficients of the DDM approximating polynomials. Mandelbrot sets were constructed for that purpose. Their use allowed receiving a more precise assessment of the world economic system stability in different years.

Keywords: Discrete Dynamic Model, GDP growth rate, growth rate sensitivity, attractive fixed points, periodic points, strange attractors, Julia set, Mandelbrot set

1. Introduction

There are various models that describe the behavior of the world economic system. Some of them explain generally how the economy functions as a whole (see Blaug, 1985) for review of these theories). Other simulate specific economic processes (Allais, 1977; Arrow, 1963; Debreu, 1970; Friedman, 1968; Harsanyi & Selten, 1988; Kantorovich, 1960; Koopmans & Montias, 1971; Lucas & Prescott, 1971; McFadden, 1986; Nash, 1951; Tinbergen, 1956; Tobin, 1955). The models describing stability of the world economic system play an important role among the latter (Ayres, 2006; Dator, 2006; Dickson, 1983; Freeman, 1987; Glazyev, 1993; Maevskiy, 1997; Mensch, 1979; Modelski, 2006; Papenhausen, 2008). The same refers to the models representing generation and development of economic crises (Marx, 1867; Khazin, 2020).

Discrete Dynamic Model used in this paper allows assessing the degree of the world economic system stability. It is grounded on the following assumptions:

1. The world economy depends on many factors $\{M\}$ that are of different nature. Consequently, growth rate of the world GDP (X) accumulates information about the influence of these factors.

2. The influence of factors $\{M\}$ on the growth rate of the world GDP has a complex and, often, non-obvious nature. It shall properly be called "influence", not "functional dependence". In

mathematical terms, such influence can be represented as mapping of *m*-dimensional space of economic parameters $\{M\}$ to the unidimensional space of the growth rate of the world GDP $\{X\}$. In the general case, the type of such mapping cannot be specified in describing economic processes; nor can its alteration as time passes.

3. World economic system is characterized by inertia, i.e. economic parameters in the year ($n+1$) "remember" their value in the year n. Consequently, growth rate of the world GDP in the ($n+1$)-th year (X_{n+1}) "remembers" its value in the n-th year (X_n). It can formally be recorded as $X_{n+1} = F(X_n)$.

Although the specific form of the function $F(X)$ is unknown due to assumption (2), it can be formally expanded in a Taylor series (1):

$$X_{(n+1)} = F(X_n) = \sum_{k=0}^{K} a_k \cdot X_n^k \quad (1)$$

where, X_{n+1} is the world GDP annual change rate in the year ($n+1$); X_n – the world GDP annual change rate in the previous n-th year; a_k – the coefficients of the expansion of the function $F(X_n)$ in a Taylor series. These coefficients can be determined by the method of least squares, by approximating actual rates of world GDP change represented in coordinates (X_{n+1}, X_n) by a polynomial of the form (1). We will call the proposed expansion the discrete dynamic model (DDM).

It should be noted that even the simplest expansion of expression (1) up to a polynomial of degree two (Chaldaeva & Kilyachkov, 2012; Kilyachkov & Chaldaeva, 2013) allows explaining all known economic cycles (Juglar, 1862; Kitchin, 1923; Kondratieff, 1935; Kuznets, 1930) from a single point of view. Expansion of expression (1) up to a polynomial of degree three (Kilyachkov & Chaldaeva, 2013; Chaldayeva & Kilyachkov, 2014) provides explanation for a complicated spectrum of rates of economic development discovered by Korotayev and Tsirel (2010).

Analysis of the properties of the approximating polynomials thus received on various time intervals brought to light a differing character of stability of the world economic system in relation to the growth rates of the world GDP. A generally accepted terminology was used to designate these stable states (Arnold, 2009; Arrowsmith & Place, 1982; Danilov, 2010; Haken, 1978; Malinetsky, 2009; Sekovanov, 2013; Trubetskov, 2010). It was demonstrated in the work (Kilyachkov et al, 2015) that stable states of the world economic system can be described as either: attractive fixed point, periodic points, or strange attractor. Each of these stable states has a radius of convergence called Julia set. Different kinds of Julia sets correspond to different types of stability (Kilyachkov et al., 2017). Comparative analysis of various types of stability and Julia sets was performed by Kilyachkov et al. (2019). Thus, Julia sets contain information on the type of stability the world economic system possesses.

However, DDM can be a means of a much deeper assessment of the world economic system sensitivity to alteration of its parameters that influence the coefficients of the approximating polynomial of DDM (1). Such analysis can be carried out by constructing a Mandelbrot set for the approximating polynomial (1) on time intervals of the corresponding Julia sets.

The Mandelbrot set is closely related to the Julia set. Namely, if the value of the coefficient of the approximating polynomial for which the Mandelbrot set is constructed, is inside the main cardioid of the Mandelbrot set, the shape of the Julia set is typical for attractive fixed points. When the value of a coefficient is moved to the edge of the cardioid, Julia set deforms, retaining connectivity. Such shape is typical for periodic points. If the coefficient values are on the edge or go beyond the main cardioid of the Mandelbrot set, the Julia set loses connectivity. This is typical for strange attractors (Sekovanov, 2013).

Thus, the position of the approximating polynomial coefficient in relation to the main cardioid of the Mandelbrot set reflects the degree of DDM stability. On the other hand, as it was noted above, coefficients of the approximating polynomial depend on macroeconomic parameters. This leads to a conclusion that the position of the approximating polynomial coefficient in relation to the main cardioid of the Mandelbrot set reflects sensitivity of the world economic system to alteration of macroeconomic parameters.

2. Method

2.1 Approach to the Problem

This work has an objective to test the sensitivity of the world economic system stability to the change of the system's parameters using Mandelbrot sets constructed on the basis of DDM. In the course of the studies conducted, the following interrelated tasks were solved:

1. Determining the coefficients $\{a_k\}$ of the approximating polynomial (1) for actual values of the rates of the world GDP change.

2. The construction of the Mandelbrot sets corresponding to $\{a_k\}$.

3. Analysis of the Mandelbrot sets thus constructed and their comparison to the results obtained earlier (Kilyachkov et al., 2019) in order to reveal peculiar properties of the world economic system stability over different time intervals.

2.2 Methods of solution

In order to solve the problem set, we used the expansion of expression $X_{n+1} = F(X_n)$ in a Taylor series up to the polynomial of degree 3, which looks as follows:

$$X_{n+1} = F(X_n) = a_0 + a_1 \cdot X_n + a_2 \cdot X_n^2 + a_3 \cdot X_n^3 \qquad (2)$$

To determine the coefficients $\{a_k\}$ of the approximating polynomial (2) based on actual values of the rates of the world GDP we used the method of least squares on a sliding interval, suggested in (Kilyachkov et al., 2019). The specific feature of the sliding interval method is that the coefficients (a_0, a_1, a_2, a_3) are determined on an interval having a fixed duration (5 years), the position of which shifts from the initial value for the considered period of time up to the final value. Thus, the coefficients (a_0, a_1, a_2, a_3) are first determined on the interval from 1961 to 1965, where the obtained values are matched to the middle of the approximation interval, i.e. to 1963, then from 1962 to 1966, and so on, till the period from 2011 to 2015, the result of which is matched to 2013. Note that the polynomial with coefficients, which correspond to the *n*-th year, approximates the behavior of the rates of the world GDP change on the interval of ($n \pm 2$) years, and its coefficients (a_0, a_1, a_2, a_3) are correspondent to some average state of the economy for this period.

In order to define the coefficients of the approximating polynomial we used the growth rates of the world GDP on the interval from 1961 to 2015 obtained from World Bank Open Data site. It allowed performing analysis of the world economic system stability in the period from 1963 to 2013.

We used a standard methodology of constructing Mandelbrot set suggested in (Sekovanov, 2013) and adapted it to DDM (Kilyachkov et al., 2018). Values of three coefficients of a polynomial (2) were fixed and equated with the polynomial coefficients, which approximated the actual data of the world GDP annual change rate for the corresponding time intervals. The value of the fourth coefficient was being altered within reasonable limits, allowing the construction of the Mandelbrot set. An expression (2) was iterated 25 times for each set of values of four coefficients $\{a_k\}$, i.e. X_{n+1}, calculated at the previous *n*-th step by formula (2), was used to calculate X_{n+2} for the next step. If the value of $| X_{n+1} |$ on 25 steps was ≤ 2, we assumed that the iterated function converges.

It is crucial to select appropriate initial values X_0 when constructing Mandelbrot set for defining convergence of the approximating polynomial (2). A natural step is to choose as X_0 GDP values that were used for calculating the coefficients of the approximating polynomial for the time interval considered. That is why five Mandelbrot sets were constructed for each of the four coefficients $\{a_k\}$ of the approximating polynomial (2), corresponding to the five years of the sliding interval from which the initial values had been obtained. Then, for convenience, these five Mandelbrot sets were depicted in the same diagram, as shown on Figure 1. The year indicated is the year from which actual world GDP growth rate was taken as initial value X_0.

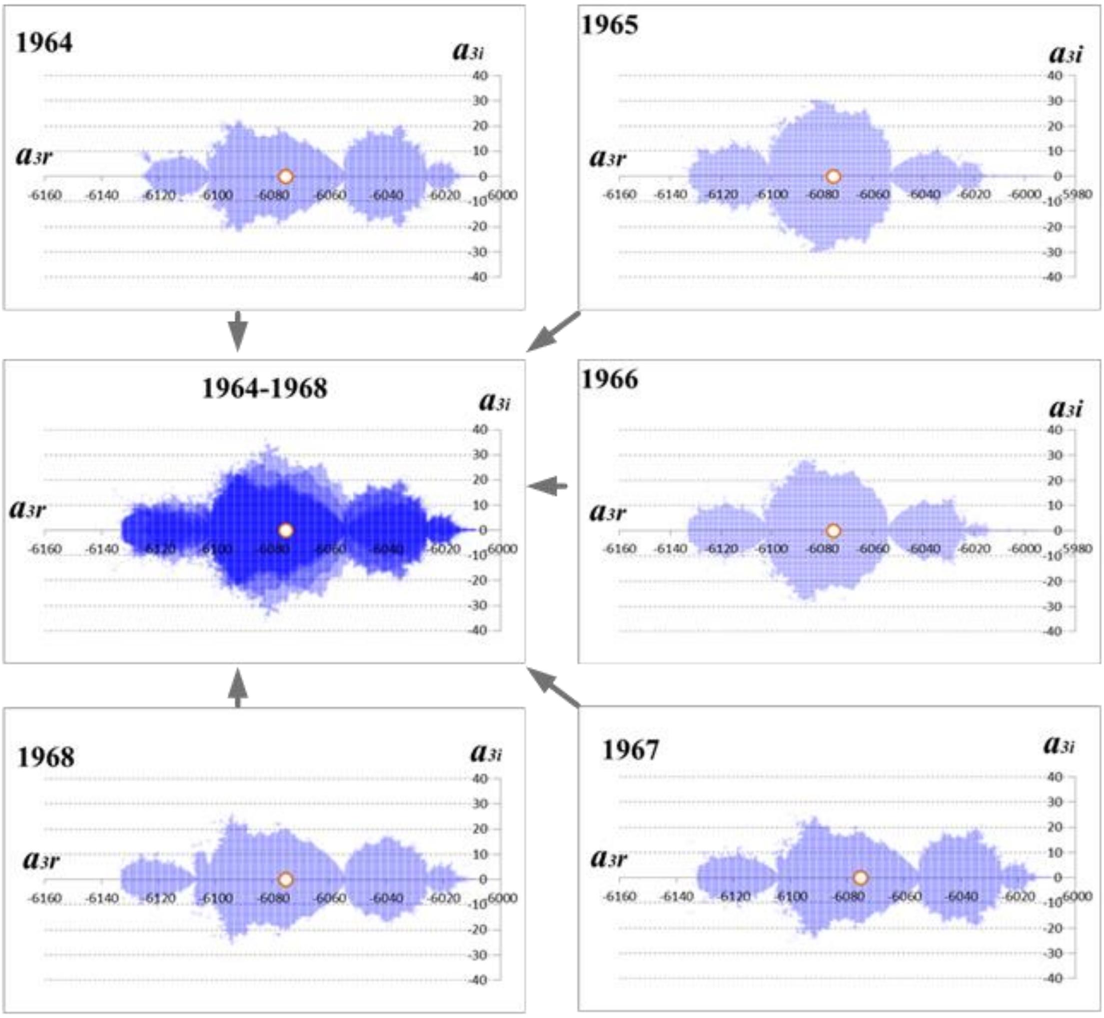

Figure 1. Methodology of construction of the resulting Mandelbrot set for 1966. Year indicated in the diagram is the year from which actual world GDP growth rate was taken as in initial value X_0 for constructing the corresponding Mandelbrot set. Circle in the diagram corresponds to the value of the coefficient of the approximating polynomial a_3 calculated using the method of least squares for the time interval selected

3. Results

In the earlier work (Kilyachkov et al., 2019), we discovered the pattern of the world economic system stability (or absence thereof) as described by DDM on the interval from 1961 to 2015. Let us use Mandelbrot sets to describe sensitivity of the world economic system growth to alteration of the system's parameter.

3.1 Stable Fixed Points

In the years (1966, 1974, 1976, 1983-1985, 2003, 2004, 2010-2013) the world economic system characterized by stable fixed points, the values of coefficients of the approximating polynomial are within the resulting Mandelbrot sets that correspond to these values. Figure 1 represents resulting Mandelbrot sets for the year 2004 constructed using different coefficients (a_3, a_2, a_1, a_0) of the approximating polynomial (2). Circle in the diagram corresponds to the value a_i of the coefficient of the approximating polynomial calculated by the method of least squares. The year indicated is the year from which actual world GDP growth rate was taken as initial value X_0.

It relates to almost all the cases characterized by stable fixed points. However, years 1974, 1976, and 1983 were somehow different. Resulting graphs for 1974 and 1976 are characterized by Mandelbrot sets spread apart into two regions (see Figure 3).

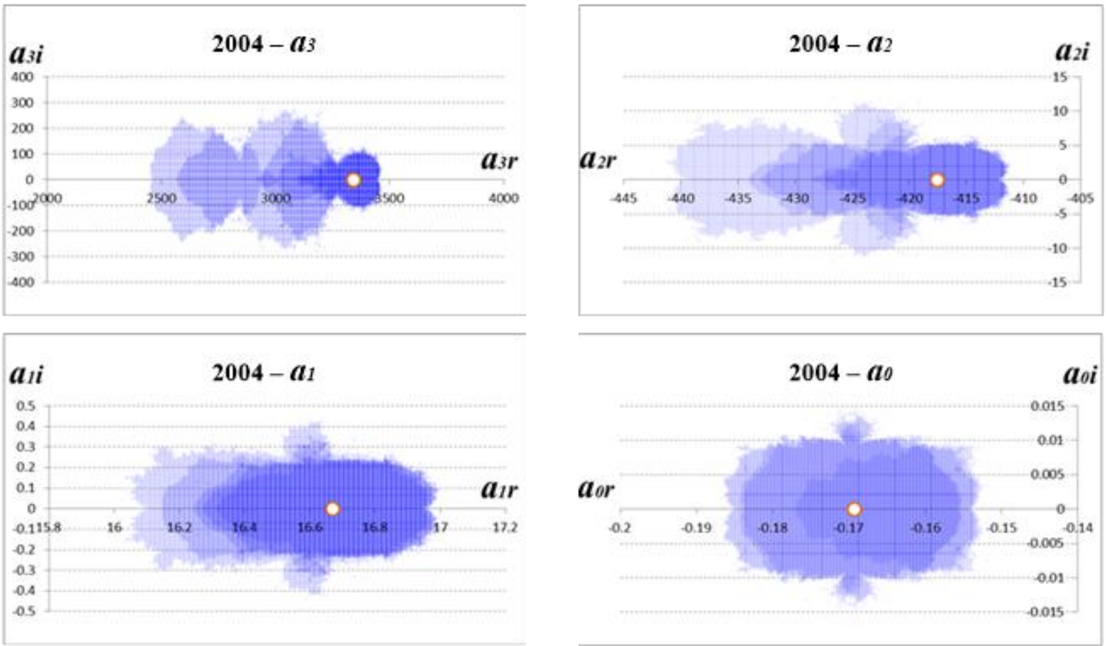

Figure 2. Resulting Mandelbrot sets for the year 2004 (stable fixed point)

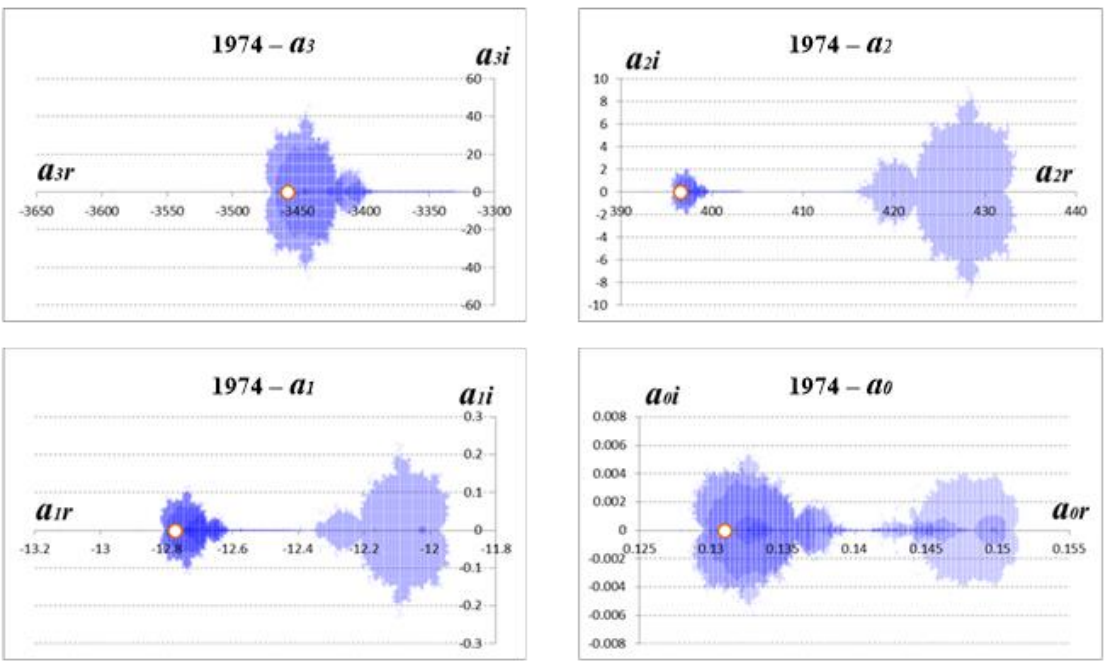

Figure 3. Resulting Mandelbrot sets for the year 1974 (stable fixed point)

This result shows that, although the world economy was in a stable state in these years, it could have switched to another stable state characterized by another Mandelbrot set if macroeconomic parameters had changed.

Analysis of Mandelbrot set for 1983 (see Figure 4) demonstrates that the pattern of the world economic stability in this year corresponds better to periodic rather than stable fixed points because the values of the coefficients of the approximating polynomial a_3, a_2 and a_1 are placed at the edge of the Mandelbrot set. The Mandelbrot set for a_0, however, corresponded to stable fixed point pattern. It could be explained by the fact that 1983 was characterized by two stable states (Kilyachkov et al., 2019). Thus, the world economic system could have switched to periodic points pattern that year if macroeconomic parameters had changed.

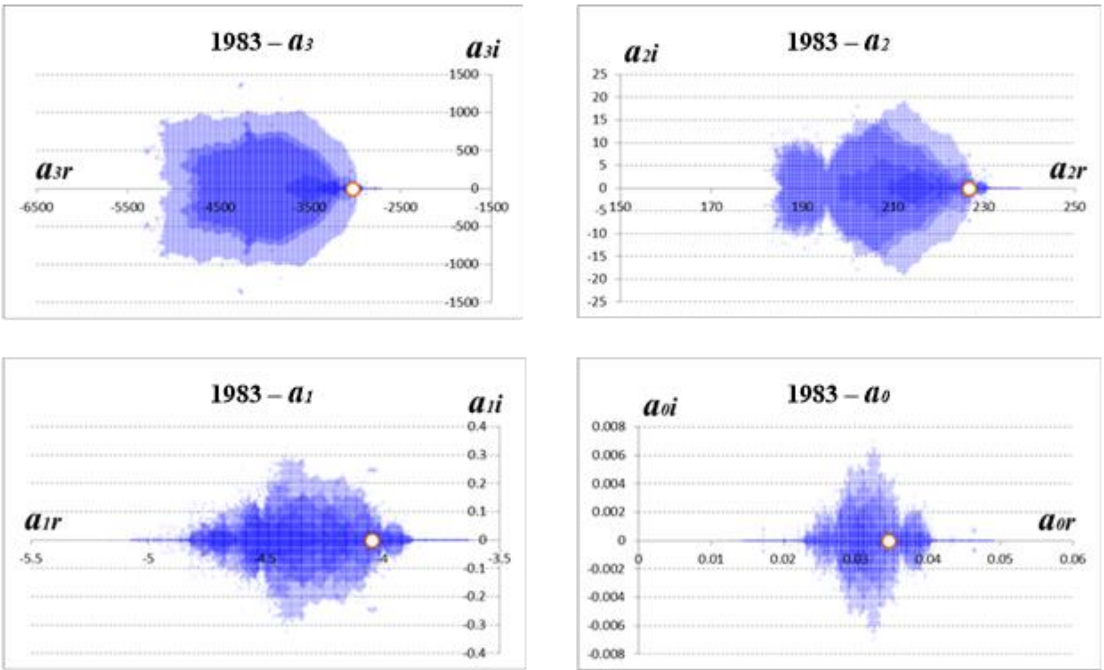

Figure 4. Resulting Mandelbrot sets for the year 1983 (a switch from stable fixed point to periodic points pattern)

3.2 Periodic Points

Coefficients of the approximating polynomials that correspond to the states of the world economic system characterized by periodic points are placed within Mandelbrot set at its edge. It occurs in all years considered (1968, 1971, 1989, 1990, 1998-2000, 2007), although some particular cases do exist.

Mandelbrot sets for 1968, 1989, 1990 and 2007 lie wholly on one side of the value of the corresponding coefficient of approximating polynomial (see Figure 5). Compact arrangement of the Mandelbrot sets indicates to a greater stability of the world economic system in the periods considered.

For 1971, Mandelbrot sets lie on either side of the value of the corresponding coefficient of the approximating polynomial (see Figure 6). It demonstrates that the parameters of the world economic system in 1969-1973 changed significantly enough to cause such an arrangement of the Mandelbrot sets.

Mandelbrot sets for 1998, 1999 and 2000 are characterized by a blurred distinction between the main and the secondary cardioid (see Figure 7). It indicates to their starting disappearance (this situation was discussed in (Kilyachkov et al., 2018). Although the values of coefficients of the approximating polynomial are placed within the corresponding Mandelbrot sets, dynamic stability of the world economic system was relatively weak in these years.

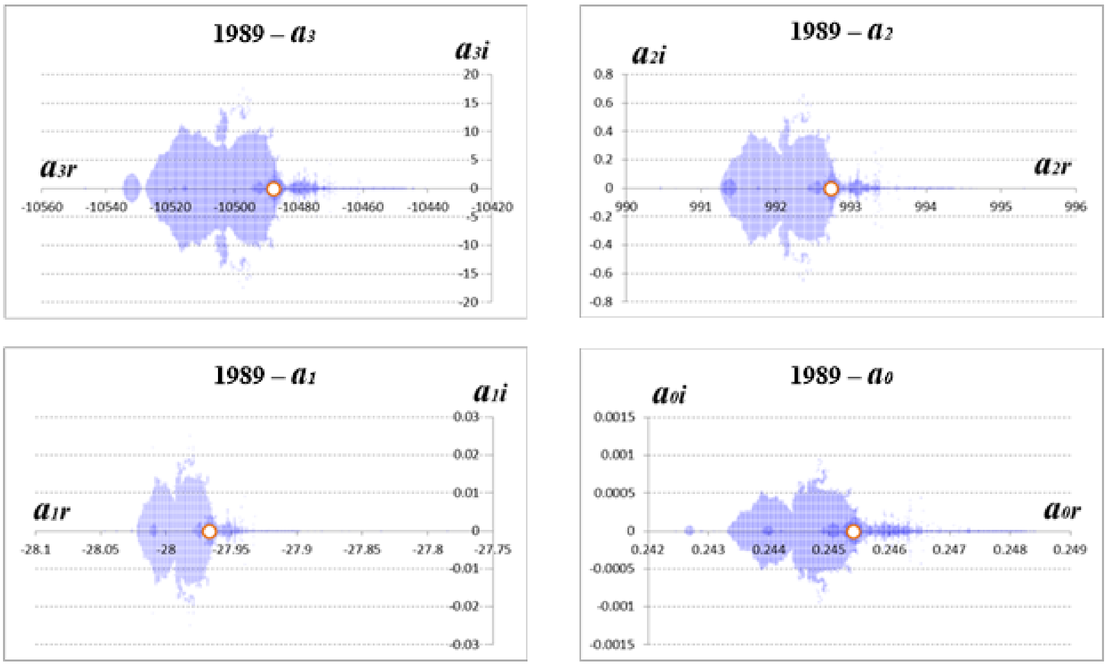

Figure 5. Resulting Mandelbrot sets for the year 1989 (periodic points)

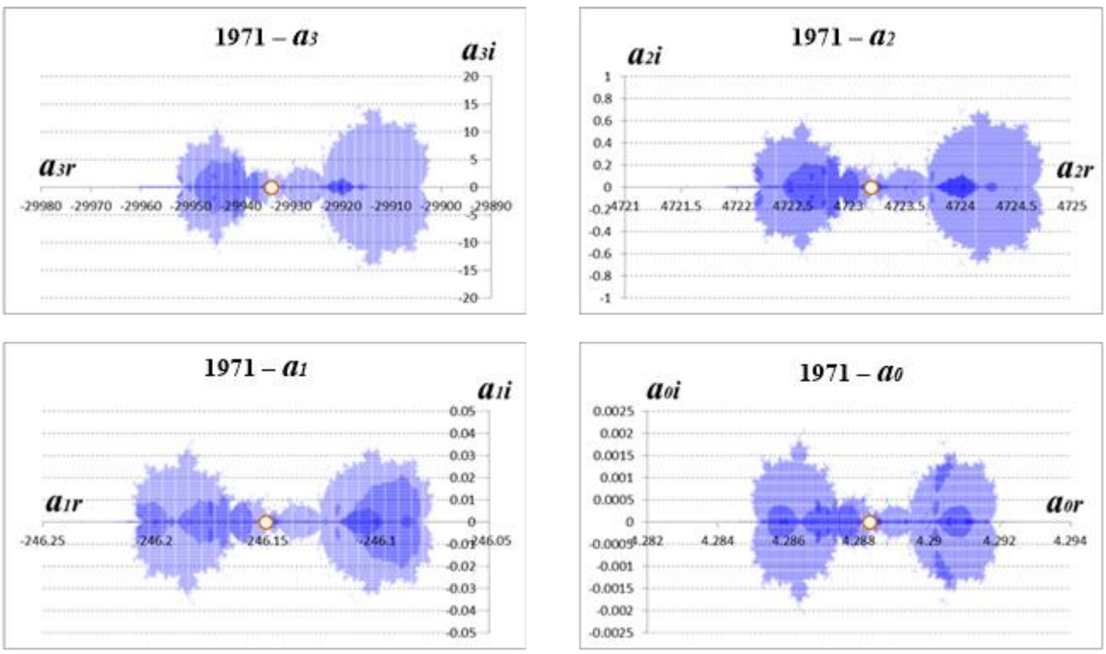

Figure 6. Resulting Mandelbrot sets for the year 1971 (periodic points)

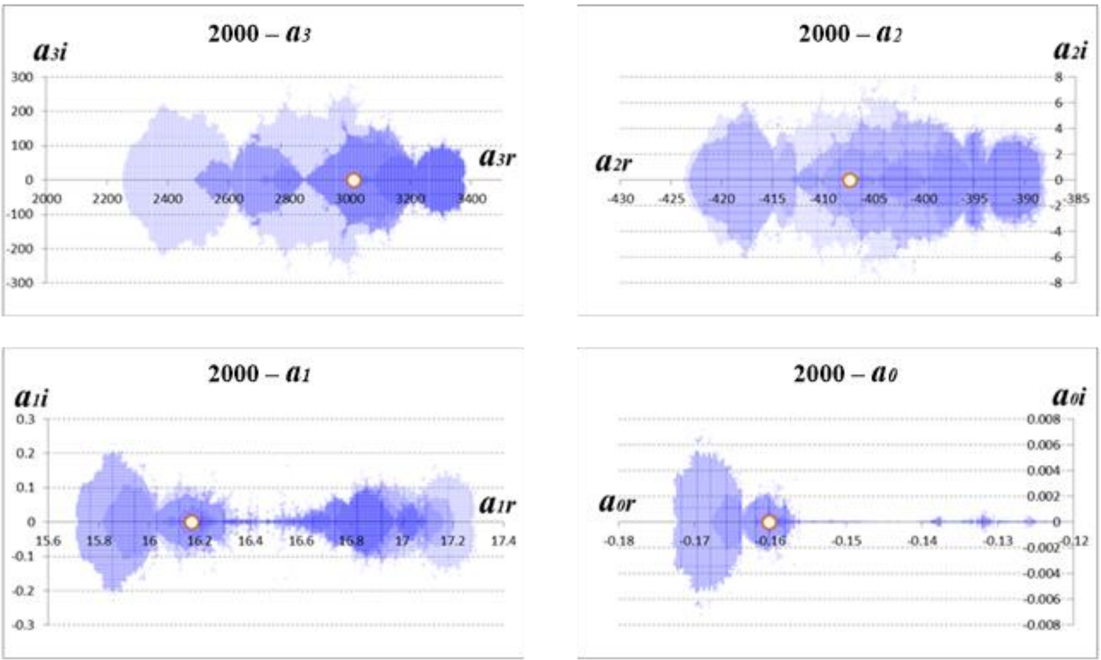

Figure 7. Resulting Mandelbrot sets for the year 2000 (periodic points)

3.3 Strange Attractors

Mandelbrot sets that correspond to strange attractors (1970, 1975, 1995-1997, 2002) lie either on one side of the coefficient of the approximating polynomial (1975, 1995 and 1996, see Figure 8) or on both sides of it (1970, 1997 and 2002, see Figure 9). In both cases the corresponding coefficients of the approximating polynomials are located at the "tip" of the Mandelbrot set.

A more compact arrangement of Mandelbrot sets in 1975, 1995, and 1996 indicates to a greater stability of the world economic system in these periods. Mandelbrot sets located on either side of the coefficient value in 1970, 1997, and 2002 show that parameters of the world economic system in these years changed significantly.

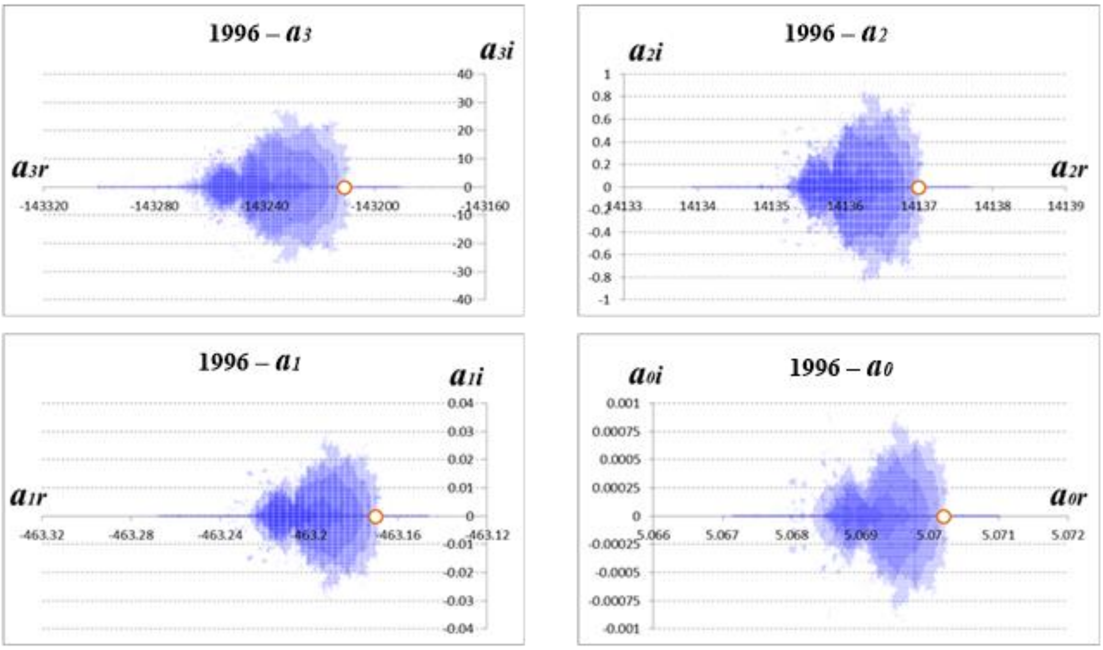

Figure 8. Resulting Mandelbrot sets for the year 1996 (strange attractors)

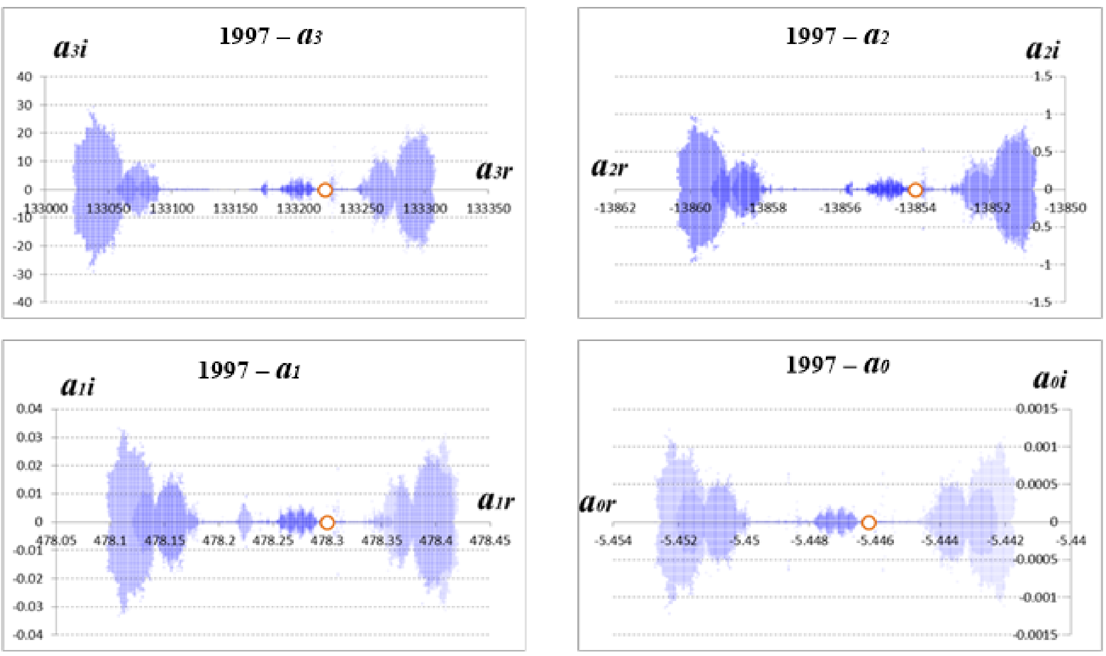

Figure 9. Resulting Mandelbrot sets for the year 1997 (strange attractors)

3.4 Basin of Attraction is Absent

As a rule, it is impossible to construct a Mandelbrot set for unstable states of the world economic system (1963-1965, 1967, 1969, 1972, 1973, 1977-1982, 1986-1988, 1991-1994, 2001, 2005, 2006, 2008, 2009). However, they do exist in some cases. And in these cases, the value of the corresponding coefficient of the approximating polynomial lies outside these sets, which can be located both on one side (1987, 1988, 1993, 1994, and 2009, see Figure 10) and on either side of it (1972, 1973, 1986, and

2001, see Figure 11). We call such states of the world economic system "quasi-stable" because the corresponding approximating polynomials diverge after 4-7 iterations.

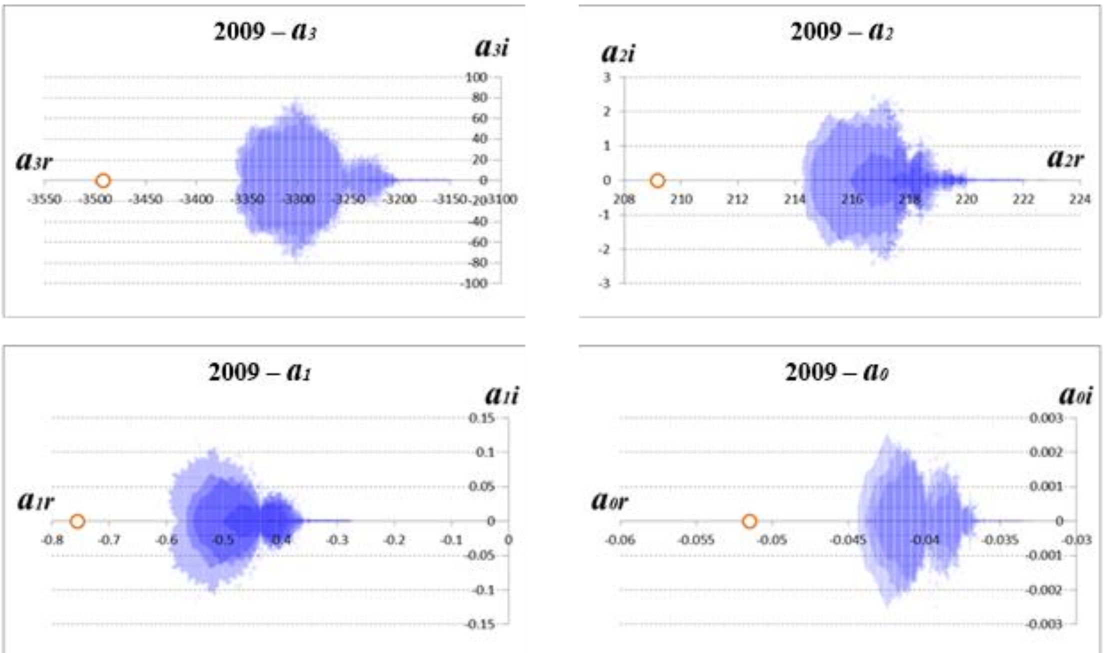

Figure 10. Resulting Mandelbrot sets for the year 2009 (basin of attraction is absent)

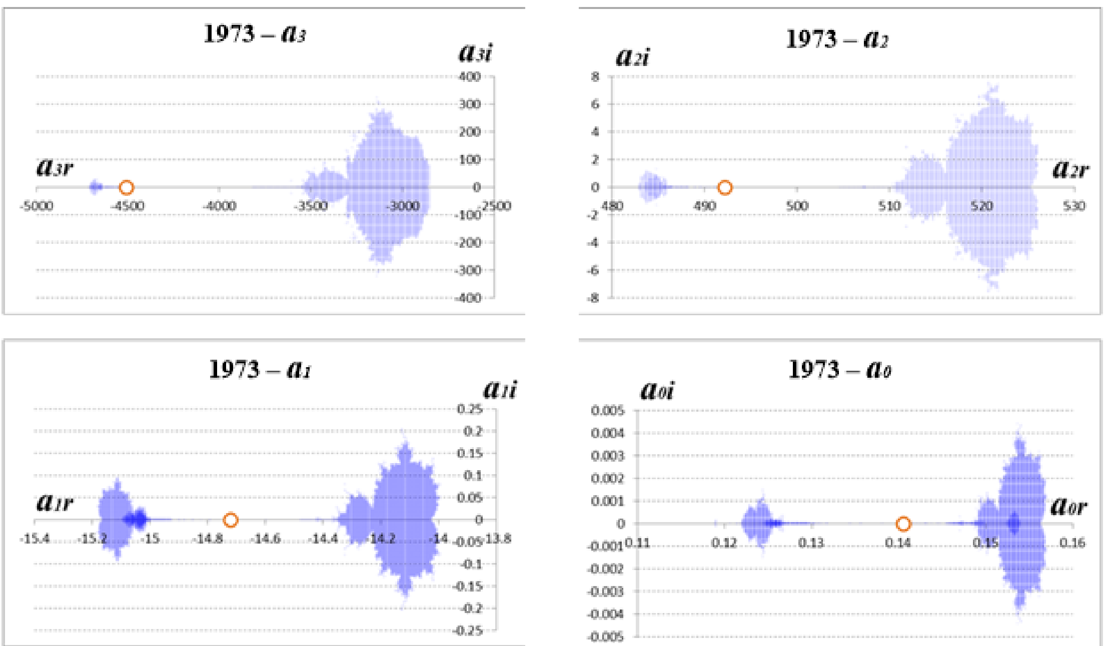

Figure 11. Resulting Mandelbrot sets for the year 1973 (basin of attraction is absent)

4. Summary

The results obtained allow constructing the following refined semi-quantitative diagram of the world economic system stability (Figure 12).

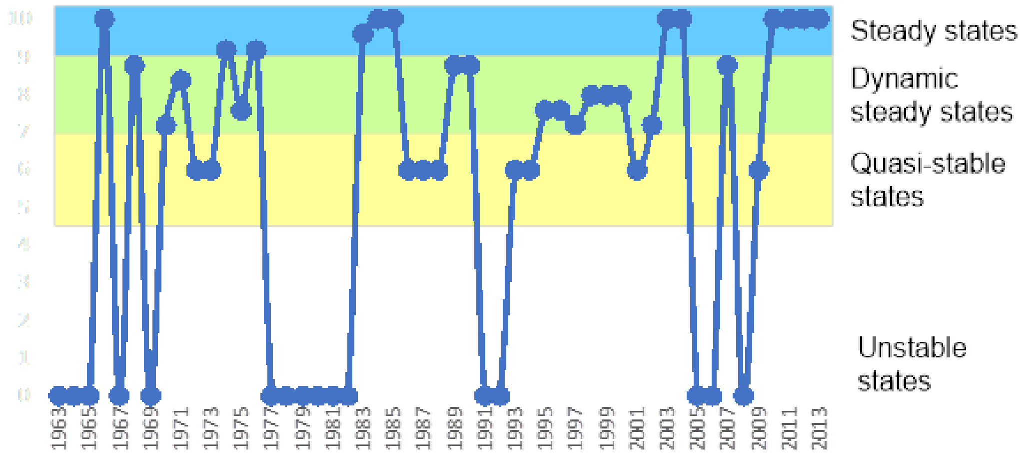

Figure 12. Semi-quantitative diagram of the world economic system stability from 1963 to 2013

Four periods of instability can be clearly distinguished in the diagram:

- 1963-1969, crisis of the world currency system;
- 1977-1982, transition from Keynesian to neo-conservative policy in the international economic relations;
- 1991-1992, re-shaping of the world economic system caused by collapse of the world socialist system;
- 2005-2008, world economic crisis.

These results confirm that the Discrete Dynamic Model adequately describes dynamics of the world economic system stability. It should be noted, however, that even during the periods of instability the world economy rose to dynamic steady or steady states in some years (or, rather, five-year periods selected under sliding scale method). The nature of such behavior in some crises (1963-1969, 2005-2008), as well as its absence during other periods of instability, is yet to be discovered.

To construct a Discrete Dynamic Model, one needs to know the values of the world GDP growth rates that are used for defining coefficients of the approximating polynomials. World Bank data are used as a source of this information. The World Bank regularly performs more precise calculations of the world macroeconomic parameters, for the latest years and retrospectively. Although the changes thus introduced are insignificant, it is still necessary to check how much the results obtained with DDM are sensitive to the retrospective amendments of the world GDP growth rates published by the World Bank.

All this work is especially important if we aim at turning DDM into a forecast instrument. Information used for it is always incomplete and preliminary. DDM could produce good forecasts if it draws attention to the first signals of future crises and is not sensitive to retrospective amendments of input data. These hypotheses should be tested.

5. Conclusion

We have studied how DDM helps to assess the stability of the world economic system (or, in other words, sensitivity of its growth to alteration of its parameters). Results obtained here, together with our earlier works, let us affirm that DDM allows:

- Explaining all known economic cycles from a single point of view;
- Explaining a complicated frequency spectrum of rates of economic development;
- Assessing stability of the world economic system in different years;
- Producing an adequate description of the world economic system stability for a period from 1963 to 2013.

The work performed revealed a need for further study of the following matters:

- Relation of different periods of instability discovered using DDM to economic crises known from economic history. Different behaviors revealed by DDM shall also be interpreted;
- Sensitivity of results obtained with DDM to the retrospective amendments of the world GDP growth rates published by the World Bank;
- Prognostic value of DDM.

Other nontrivial results that might be obtained with DDM should also be marked, described and interpreted.

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