Original Paper

Minimization of Actuarial Gains and Losses and Annual Expenses for Next Year

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Abstract

In actuarial valuations, one of the fundamental objectives is the minimization of actuarial Gains and Losses, in order to impact as little as possible the equity account in OCI, (OTHER COMPREHENSIVE INCOME STATEMENT). Under IAS 19, actuarial gains/losses cannot be amortized over the average future working life of the population.

Minimizing losses is not an easy task, given, among other things, the uncertainty of the expected benefits to be paid, discount rates and salary increases, as well as staff turnover rates.

In this paper, the Actuarial Gain / Loss (G/P) is modeled when salary increase rates vary as a **decision variable** that in some way affects the Actuarial Liabilities of the fiscal year (PBO) and therefore the next year annual cost.

Keywords: IAS19, Actuarial Losses, Optimization process, Projected Unit Benefit Method, Social Benefits, Hyperinflation, Decision variables, net periodic annual cost

JEL: C65, G23, J64

1. Background

In Venezuela, the levels of actuarial losses are generally high, fundamentally due to the inflationary component, salary adjustments are very varied, impacted by high inflation and changes in the minimum wage promoted by government labor policies, which in some way displaces up the different salary levels of the company. All of the above justify the need to model both the actuarial loss the actuarial liability and next year's expense.

2. Objective

Modeling the projected benefit obligation (PBO) via optimization, trying to minimize the value of the actuarial loss or gain, and the prediction of next year's expense.

3. Actuarial Model

In hyperinflation, real rates are generally used to carry out actuarial valuations or proxies, such as JP Morgan's EMBI, plus an American risk-free rate, as a reference. Then the total yield of the government bonds of a Latin American Country would be given by $R = TLR + EMBI$, the above, would be easily explained as that minimum discount rate that an investor would expect, to be able to invest in a country and/or eventually finance a project or funding of a defined benefit fund for the employees of a certain company. The above in foreign currency, to take them to a functional unit of the country, a transformation of the previous amount should be made, adjusted for inflation in both countries and for the devaluation rate against the dollar.

UNIT CREDIT PROJECTED BENEFIT METHOD

The model used to determine the **Actuarial G/P** from the movement of the obligation is described in detail below.

a. **The dynamics of the Actuarial Liability is determined as follows for a fiscal period:**

$$PBO^J_{t+1} = PBO^J_t + CS_{t,t+1} + CI_{t,t+1} - B_{t,t+1} + {}^G/_P \qquad (1)$$

CLEARING OUT ACTUARIAL GAIN/LOSS

$$\Delta PBO^J_t = PBO^J_{t+1} - PBO^J_t$$

$$\left({}^G/_P\right)_{(t,t+1)} = \Delta PBO_T - CS_{t,t+1} - CI_{t,t+1} + B \qquad (2)$$

$CS : SERVICE\ COST\ (t, t+1)$: Represents the change in the actuarial obligation for one more year of service and salary.

$CI : INTEREST\ COST\ (t, t+1)$: Represents the interest generated by the liability (product of the interest rate times the respective PBO).

$B : PAID\ BENEFITS\ (t, t+1)$: Benefits paid to employees.

$PBO^J_t : ACTUARIAL\ OBLIGATION\ INT\ FOR\ j$

$j : \{VS, NS\}\{VD, ND\}$

$VD : OLD\ DATA$

$ND : NEW\ DATA$

$VS : OLD\ ASSUMPTIONS$

$NS : NEW\ ASSUMPTIONS$

key and critical aspects in an actuarial valuation is precisely to define a set of both demographic and economic hypotheses. In our case to carry out the actuarial valuation, the following actuarial assumptions were used:

i)Real discount rate: 4%

ii)Nominal salary increase rate: 700%

iii)Nominal nominal interest rate (discount):732% (Note 1)

iv)Turnover rate by age: Company experience

v)Mortality rate by age: GAM 83

The results of this valuation with these assumptions, generated a relatively high loss with an annual expense for the next year also high, as can be seen later on.

b. **Assumptions and actuarial hypotheses of the optimization:**

The fundamental objective is the minimization of the actuarial Gains / Losses, trying at the same time to minimize the annual expense of the following year through the minimization of the cost of interest.

$$E(PBO^{vs}_t) : Expected\ Liability \qquad (3)$$

$$PBO^{vs}_{t+1} - E(PBO^{vs}_t) = Experience\ actuarial\ liability \qquad (4)$$

The Actuarial G/P implies minimizing the Actuarial Liability PBO, but the latter competes with the expense of the year; because the aforementioned relationships operate in the opposite direction, that is, the higher the real interest rate, the lower the liability but the higher the expense for the next year.

$$a > i_R = \downarrow PBO \uparrow GA \qquad (5)$$

Real Interest	PBO_{t+1}	GA_{t+1}
High	↓	↑
Base	-	-
Low	↑	↓

(6)

c. Determination of the impact of the Defined Benefit Plan due to changes in actuarial assumptions and hypotheses.

$$\Delta PBO_t^{ns-vs} = PBO_{t+1}^{ns} - PBO_t^{vs} \tag{7}$$

ΔPBO_t^{ns-vs}:Differential due to change of assumptions

As is well known, part of the actuarial G/P is due to 2 components, the experience of the plan explained by (4) and the change in assumptions explained by (7).

4. Data Base

The statistics of the sample used based on the following variables are described below:

i)**Population:** # employees

ii)**Current Age:** Average Age

iii)**Current service:** Average Seniority

iv)**Comprehensive salary:** Average reference salary

v)**Payroll:** Monthly value of the payroll

Broken down by gender and type of administrative or confidential payroll.

Table 1. Active employees

	ITEM	Employees					
		Adminitrative			Confidential		
		FEMALE	MALE	TOTAL	FEMALE	MALE	TOTAL
Statistics	Population	18	16	34	17	12	29
	Average Age	45,19	47,45	46,25	43,06	44,37	43,60
	Average Service	7,71	7,45	7,59	3,54	1,08	2,52
	Integral Salary	522,73	467,46	496,72	816,75	916,68	858,10
	Average	9.409,13	7.479,34	16.888,47	13.884,75	11.000,16	24.884,91

This is a total of 63 employees discriminated by payroll and gender.

5. Results under Ias-19 of the Model Used in the Valuation

Based on all the assumptions and hypotheses of salary increase, real and nominal interest rates, together with demographic, turnover and mortality rates, it is found that the liability in $(t + 1)$ is (14,113.97) an actuarial gain instead of the predicted loss and expense next year is in the order of 984,906.36. When the results of the initial assessment are compared with the optimized one, it is found:

Table 2. Table Optimization Results

Results Under NIC - 19	Initial Results	Optimized Results	Impact of optimization
G/P Actuarial	10.111,69	-14.113,97	Changed from a Loss to a Profit
PBO	155.610,72	131.385,96	Lowered the actuarial liability
GA	1.186.723,20	984.906,36	Reduced annual spending

G/P: Actuarial Gains / Loss
PBO: Actuarial Liability
GA: Annual Expense

RESULTS OF THE OPTIMIZED ACTUARIAL VALUATION

Table 3. Reconciliationn of Liabilities/Assets

CHANGE OF LIABILITY	
LIABILITIES	
1) (PBO) Beginning	**9.844,92**
2) Interest Cost	**121.265,27**
3) Service Cost	**15.619,53**
4) Personal Reserve Transfered	-
5) Past Service Cost	-
6) Paid Benefits	**(1.230,68)**
7) Gain / Loss Actuarial	(14.113,97)
8) (PBO) Ending	131.385,06

FINANCIAL STATEMENT FOR LIABILITIES	1	
1) (PBO) Liabilities	131.385,06	
2) Book Reserve	-	
3) Gain / Losse Actuarial	131.385,06	
4) Past Service Cost	131.385,06	
5) Recognized liability on balance sheet for the company	-	
6) Gain / Losse Actuarial	(14.113,97)	
ANNUAL COST	**2020 - 2021**	**2021 - 2022**
1) Services cost	15.619,53	128.195,18
2) Interest cost	121.265,27	856.711,18
4) Increase in the Obligation for migration	-	-
5) Past service cost	-	-
6) Settlement	-	-
5) Expense recognized in the Company's Income Statement	136.884,80	984.906,36
6) Annual Payroll Assets (5)	43.285,40	595.536,19
7) Payroll cost as a percentage	316,24%	165,38%

The foregoing is undoubtedly of great importance for the sustainability of the benefit plan and of the companies, which in some way are greatly affected by the hyperinflationary atmosphere in which they operate.

6. Results of the Optimization Based on the Assumptions

The minimization of the actuarial loss by $PA(t, t+1)$ controlling the decision variable salary increase rate in the domain $650\% \leq TAS \leq 750\%$, with an incremental step of 5% and its real equivalents yielded the following:

Table 4. Sample of Solutions

		Objective	Restrictions
		Minimize final value	Final value <= 15.000,00
Simulations	Solution Number	5) Expense recognized in the Company's Income Statement 2021 - 2022	7) Actuarial loss (gain) on obligation · 1
1	8	984.906,36	(14.113,97)
2	11	1.046.226,72	(6.744,17)
3	17	1.113.279,39	1.305,41
4	1	1.186.723,20	10.111,69
5	16	928.734,87	(20.872,23)
6	14	1.267.303,05	19.761,49
7	4	877.194,90	(27.079,56)
8	15	829.827,16	(32.789,84)
9	12	786.223,38	(38.051,06)
10	9	1.355.862,13	30.352,94
11	6	746.020,20	(42.906,00)
12	21	708.893,78	(47.392,83)
13	2	674.555,17	(51.545,67)
14	13	1.453.356,16	41.997,04
15	20	1.560.869,63	54.819,57
16	5	1.679.634,79	68.963,17
17	18	1.811.053,36	84.589,80
18	3	---	---
19	7	---	---
20	10	---	---
21	19	---	---

From the table above, it can be inferred that the minimum final value of the simulations for the annual expense was in the order of 984,906.36 and the corresponding actuarial gain was (14,113.97).

The previous solution corresponds to iteration #8 of the 21 solutions evaluated.

Table 5. Actuarial Losses/Gains

Final value >= (20.000,00)	Constraints	
7) Actuarial loss (gain) on obligation · 1	Report NIC-19'!B30 <= 'Report NIC-19'!B31	Report NIC-19'!C59 <= 'Report NIC-19'!B59
(14.113,97)	-14.113,97	1,65
(6.744,17)	-6.744,17	1,76
1.305,41	1.305,41	1,87
10.111,69	10.111,69	1,99
(20.872,23)	-20.872,23	1,56
19.761,49	19.761,49	2,13
(27.079,56)	-27.079,56	1,47
(32.789,84)	-32.789,84	1,39
(38.051,06)	-38.051,06	1,32
30.352,94	30.352,94	2,28
(42.906,00)	-42.906,00	1,25
(47.392,83)	-47.392,83	1,19
(51.545,67)	-51.545,67	1,13
41.997,04	41.997,04	2,44
54.819,57	54.819,57	2,62
68.963,17	68.963,17	2,82
84.589,80	84.589,80	3,04
---	166.015,26	4,20
---	121.054,26	3,56
---	101.883,62	3,29
---	142.340,62	3,86

The minimization objectives were the actuarial loss and the next year's expense. This was achieved by imposing 4 requirements and 2 restrictions with 1 decision variable.

After carrying out the 21 solutions evaluated, the final value of the expense recognized for the next year in the income statement went from 1,186,723.20 to 984,96.36, a change of 17.01%.

Within the optimization requirements and constraints block are:

1)   The actuarial gain or loss must be equal to or greater than (-20,000 Bs) and in turn less than the actuarial liability for the fiscal period, that is, $\left(G/P\right)_{t,t+1} \leq PBO_{t+1}$

2)   The actuarial gain or loss must be less than 15,000 Bs.

Table 6. Expenses

Objective	Restrictions	Decision
Minimize final value	Final value <= 15.000,00	variables
5) Expense recognized in the Company's Income Statement 2021 - 2022	7) Actuarial loss (gain) on obligation · 1	s · 3
984.906,36	(14.113,97)	685,00%
1.046.226,72	(6.744,17)	690,00%
1.113.279,39	1.305,41	695,00%
1.186.723,20	10.111,69	700,00%
928.734,87	(20.872,23)	680,00%
1.267.303,05	19.761,49	705,00%
877.194,90	(27.079,56)	675,00%
829.827,16	(32.789,84)	670,00%
786.223,38	(38.051,06)	665,00%
1.355.862,13	30.352,94	710,00%
746.020,20	(42.906,00)	660,00%
708.893,78	(47.392,83)	655,00%
674.555,17	(51.545,67)	650,00%
1.453.356,16	41.997,04	715,00%
1.560.869,63	54.819,57	720,00%
1.679.634,79	68.963,17	725,00%
1.811.053,36	84.589,80	730,00%

7. Conclusions and Recommendations

1.   As we have already seen the issue of **minimizing actuarial losses** subject to next year's annual expense also being as low as possible, it is not an easy task.

2.   The choice of assumptions and actuarial hypotheses are in turn very restricted by the economic situation of the country, in which case, more than specific estimates of the nominal rates of salary increase and interest, small ranges of variability obviously of amplitude, should be had. not too big to be able to find a feasible solution that, as far as possible, falls within the intervals chosen based on responsible management criteria where reasonableness prevails, in the space defined by the pair (salary, interest).

3.   In countries with high inflation, such as Venezuela, it is almost a **Desideratum** to carry out optimization processes that go towards achieving this objective.

4.   Lowering liabilities and simultaneously spending is not an easy task, because both variables exhibit antagonistic behavior. However, as already stated in the results, it was possible to go from an actuarial loss to a gain, which improves the accumulated position in the **equity account** for this concept, and a decrease in the annual expense for the following year, improving the **state of results**.

5.   In countries with high inflation, where the number of percentage digits is 3 or more, as in the Venezuelan case, it is very important to define the space generated by inflation, the salary increase rate and the interest rate to discount obligations, given orders of magnitude, which are observed in these economies.

6.   **The optimal salary increase rate was 685%**, which falls within the assumed variability range (650%, 750%), for setting optimization assumptions.

7.   The main contribution of this work is to explore alternative scenarios, within a reasonable margin of variation of the Salary Increase Rate granted by the company, limiting a little the variability levels of the actuarial Gain/Loss to the lowest possible predictive expense of the next year.

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Note

Note 1. The Nominal interest rate to discount the Liabilities is generated via FISHER adjusting the salary increase rate with the real interest rate.

Annex I

The descriptive statistics of the optimization are summarized below, indicating in each case the objective, restrictions and requirements.

Table 7. Optimization Goals

Statistics	Objective	Restrictions
	Minimize end value	Final value <= 15.000,00
	5) Expense recognized in the Company's Income Statement 2021 - 2022	7) Actuarial loss / gain in financial statment
Minimum	674.555,17	(51.545,67)
Average	1.118.274,37	1.788,57
Maximun	1.811.053,36	84.589,80
St. Dev	353.565,83	42.369,16

	Restrictions		Decision variables	
Final value >= (20.000,00)				
7) Actuarial loss (gain) on obligation 1	NIC-19 Report'!B30 <= 'NIC-19 Report'!B31	NIC-19 Report'!C59 <= 'NIC-19 Report'!B59		Salary increase rate
(51.545,67)	-51.545,67	1,13	650,00%
1.788,57	26.747,60	2,23	700,00%
84.589,80	166.015,26	4,20	750,00%
42.369,16	65.806,13	0,93	31,02%

Goals	Best Solution	
Minimize the Final Value of 5) Expense recognized in the Company Income Statement 2021-2022	984.906,36	Cell: C57
Requirements		
The final value of 7) actuarial loss (gain) in Obligation 1 must be less than 15,000.00	-14.113,97	Cell: B30
The final value of 7) actuarial loss (gain) in Obligation 1 must be greater than (20,000.00)	-14.113,97	Cell: B30
Restrictions	**Left side**	**Right side**
1 IAS-19 B30 Report = IAS 19 B31 REPORT	-14.113,97	131.385,06
2 IAS-19 C59 Report = IAS 19 B59 REPORT	1,65	3,16
Decision variables	**Best solution**	
	685,00%	Cell: C6

Annex II

ACTUARIAL VALUATION MODEL

Organization chart 1