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Original Paper

A Practical Approach to the Approximate Determination of Actuarial Gains and Losses on the Basis of Aggregates

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Abstract

This paper discusses actuarial gains and losses differentiated by their aggregate-level components, i.e., Pension Benefit Obligations (PBOs) of actuarial valuations. One of the most important aspects when evaluating a defined benefit plan is to determine the magnitude of the actuarial gains/losses and to know the origin or source that generates them. The idea is to break down the actuarial total Gain or Loss into its parts. It is important to know how much corresponds to the experience of the plan and regarding the change of assumptions, in turn, in the latter it is important to know the changes in the economic part and in the demographic part. In the change of economic assumptions, the effect of the rate of change of wages and that of the rate of change of interest can be decomposed to discount obligations.

Keywords: Projected Profit Method, Actuarial Gains and Losses, Plan Experience, Actuarial Assumptions and Assumptions, Movement of Obligations Under Projected Unit Credits (PUC)

1. Context and Background

Over the last 5 years Venezuela has suffered from hyperinflation, forcing change in the assumptions and actuarial hypotheses to reflect the reality of the economic situation of the country, especially with respect to the real increase in wages.

In the last 2 or 3 years, the economy has been gradually improving a little, and the high rates of wage increases (4000%) and devaluations (200%) while still high, have been falling, which has forced adjustments to nominal rates downwards, such as from 1000% to 100%, for wage increases.

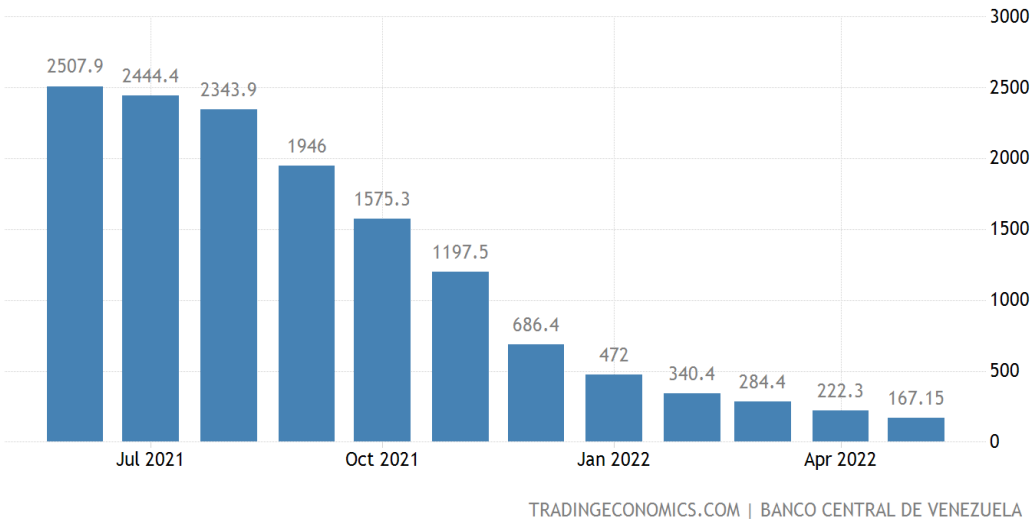


Chart 1

Generally, the rates to discount obligations are in the order of 2% on nominal inflation and an equivalent of 4% on wages, in real terms. According to the Fisher Effect, the real interest rate to discount the obligations is the quotient of interest rates and nominal wages minus one. This theory is describing the relationship between both real and nominal interest rates, and inflation. The theory states that the nominal rate will adjust to reflect the changes in the inflation rate for products and lending avenues to remain competitive.

Namely,

$$Interest\ Nominal\ Rate = (Salary\ Increases\ Nominal\ Rate\ +\ 1)\ (1 + Real\ Rate)$$

All the above sometimes generates actuarial losses in the results of the Actuarial Liabilities of the actuarial valuations of any fiscal period, which are not easily predictable, but should be explainable. The drastic changes of going from a nominal salary increase rate, for example, from 2000% to 100%, trigger very important distortions when determining the different impacts of changes in salary and interest rate increases.

Making detailed calculations of actuarial gains/losses can become very cumbersome even with sophisticated computer systems. In this sense, in this work the orientations of Josiah Lynch (Transactions of Actuaries, 1975) are partially followed.

2. Objective

The purpose of this study is to develop a model to be able to give a reasonably acceptable approximation of actuarial gain or loss discrimination in a tax period based on the experience of the plan and the change of actuarial assumptions. This will advance the capability of pension actuaries to project future obligations and provide various stakeholders more confidence in the ability to provide promised benefits to qualified recipients.

3. Mathematical Model

The mathematical model used is then developed and a movement of the obligation of a fiscal period is illustrated, considering the elements that make it up. It starts from the initial obligation of the liability, adding the cost of service and the cost of interest, less the benefits paid in the period, and from there with the final actuarial liability the profit or loss of the period is determined.

The methodology used to carry out this actuarial point of view study corresponds to the projected unit credit method (PUC METHOD) known as the **UNIT CREDIT PROJECTED METHOD.**

The model used to determine actuarial Gains/Losses (G/L) from the movement of the obligation is developed in detail below.

a. The dynamics of actuarial liabilities are determined as follows

$$PBO_{t+1}^{J} = PBO_{t}^{J} + CS_{t,t+1} + CI_{t,t+1} - B_{t,t+1} + G/L$$

$CS: SERVICE\ COST\ (t, t+1)$

$CI: INTEREST\ COST\ (t, t+1)$

$B: PAID\ BENEFITS\ \ (t, t+1)$

$PBO_{t}^{J}: ACTUARIAL\ LIABILITIES\ IN\ t\ BASE\ IN\ j$

J: {VS, NS} X {VD,ND}

VD: OLD DATA.

ND: NEW DATA.

VS: OLD ASSUMPTIONS.

NS: NEW ASSUMPTIONS.

$$\Delta PBO_t^J = PBO_{t+1}^J - PBO_t^J$$

$$\left({}^{G}/_{P}\right)_{(t,t+1)} = \Delta PBO_t - CS_{t,t+1} - CI_{t,t+1} + B$$

a. Determination of the impact of the Defined Benefit Plan by experience:

$$E(PBO_t^{vs}): Experience\ Actuarial\ Obligationsliability$$

Based on old assumptions with annual service cost and interest.

$$VS: OLD\ ASSUMPTIONS\ EPBO_t^{vs} = C_{t,t+1}^{vs} + C_{t,t+1}^{vs}$$

$$VS: OLD\ ASSUMPTIONS\ \Delta PBO_t^{experiences} = PBO_{t+1}^{vs} - EPBO_t^{vs}$$

c. Determination of the impact of the Defined Benefit Plan due to changes in assumptions and actuarial hypotheses.

$$\Delta PBO_t^{ns-vs} = PBO_{t+1}^{ns} - PBO_t^{vs}$$

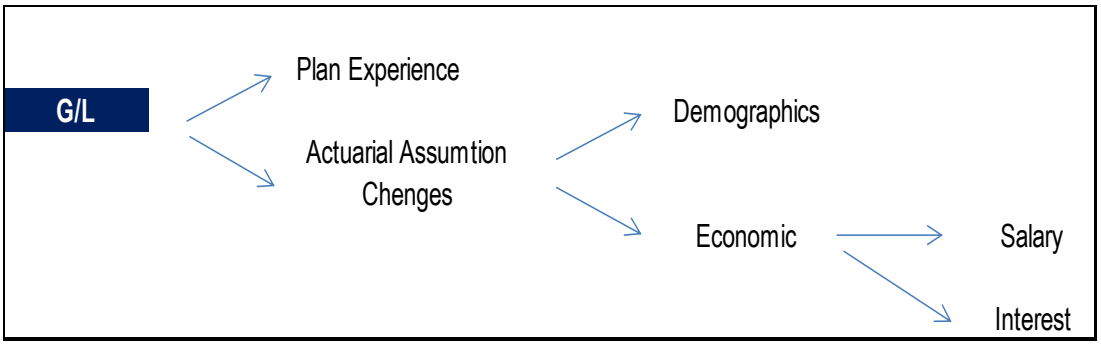
$$(G/P)_{t,t+1}^{\Delta i} = PBO_{t+1}^{(NS\Delta i)} - PBO_t^{vs}$$

VS: OLD ASSUMPTIONS.

NSΔi: CHANGE OF THE NEW INTEREST RATE.

d. Determination of the impact of the change in the interest rate or the salary increase rate keeping the rates constant with the old assumption.

At a conceptual level, the previous formulations from equations (1) to (9) correspond to the tree that discriminates the total loss into its basic components.



4. DATA

The data with which we worked in this article is described below from the demographic point of view in terms of age, the years of service of each employee and the corresponding integral salaries for purposes of calculating the social benefits that are paid in Venezuela.

		ACTIVE			2330,00%	
		TOTAL			S=2330% i=4% real R=35%	
	CONCEPT	FEM	MAS	TOTAL	30/12/2020	DIF 2021/ 2020
Statistics	Population	63	113	176	166	6,02%
	Actuarial Age (Average)	40,51	41,29	41,01	42,92	-4,43%
	Current Service (Average)	4,35	6,01	5,42	8,22	-34,10%
	Comprehensive Salary (Average) (Bs.)	1.092,59	700,48	840,84	89,76	836,78%
	Payroll / Month (Bs.)	68.833,09	79.154,56	147.987,65	14.899,95	893,21%

From the table above, it is easy to see that the increase in the average monthly salary rose 838.78%, instead of the assumption of 2,330%.

A total of 176 employees are observed, with an average age of 41.01 years, an average seniority of 5.42 years and an average monthly salary of 840.54 Bs. The benefits to be paid by the plan are generally of a retroactive and are determined based on the integral salary and the years of service of the type.

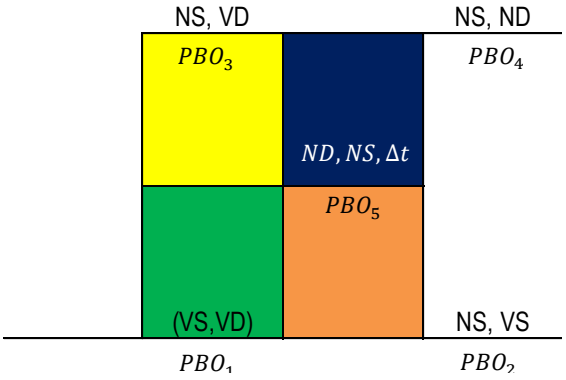
$$B_t = (SERVICE_t)(AVERAGE\ COMPREHENSIVE\ SALARY_t)$$

5. Scenarios Evaluated

To arrive at the decomposition of actuarial gains and/or losses between, the following simulations were performed:

Each scenario is defined by the changes observed in the data (VD,ND) and in the assumptions (VS,NS), that is, the change in the respective PBOs is compared using the new data (ND) with new assumptions in (t + 1) versus the old data, with the old assumptions in (t).

In order to know all the sensitivities of the PBO based on the Cartesian product of the pairs (VS,NS) X (VD,ND), the 5 scenarios of the graph below were generated:

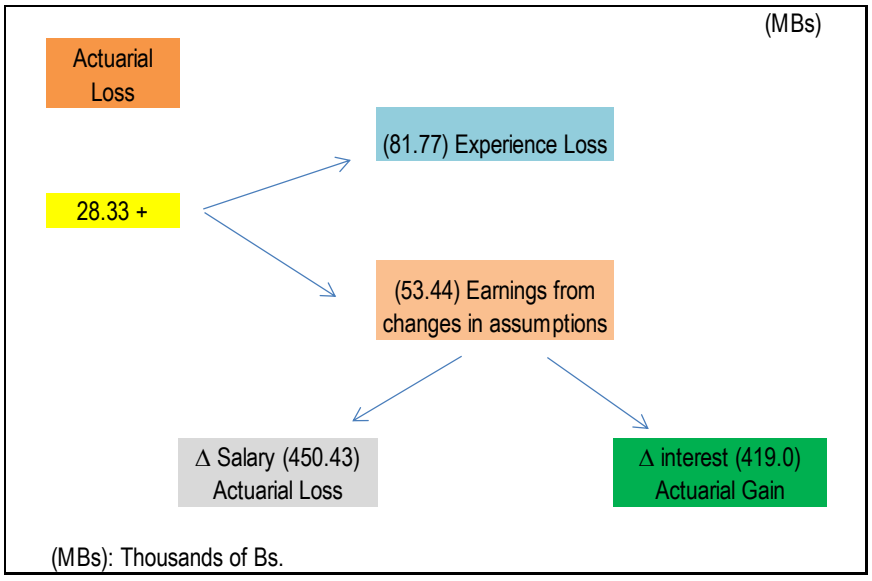


#	SCENERY	DESCRIPCTION
1	VS,VD	Data and assumptions in the initial assessment
2	VS,ND	New data without change of assumptions
3	NS,ND,Δ i	New Data New Assumptions Changing only the interest rate
4	NS,VD	Old data with new assumptions
5	NS,ND,Di	Data and assumptions in the final valuation

6. Results of the Scenarios Generated

Below are the results obtained according to the actuarial model used and the actuarial method of projected unit credit benefits.

SCENARIOS	DATA	ASSUMPTIONS	TOTAL PBO	G/P
1	DATA 2021	New Assumptions	450.432,84	28.337,56
2	DATA 2021	Old Assumptions	503.878,35	81.783,07
3	DATA 2021	Mixed Assumptions	31.436,05	-390.659,22
4	DATA 2020	New Assumptions	66.549,71	67.114,73
5	DATA 2020	Old Assumptions	75.262,66	75.827,69



DEVELOPMENT OF ACTUARIAL (G/L) DECOMPOSITION INTO (MBs.)

1) $PBO_{real}^{VS}\ (t+1) = 503.87 \quad SV$

2) $PBO_{real}^{NS}\ (t+1) = 450.43 \quad SN$

3) Assumed Changes: (53.44) (2-1)

4) Actuarial loss for the year: 28.33

5) Plan Experience: 81.77 (4+3)

6) $PBO_{t+1}^{NS}\ changing\ i: 31.43 \quad SNI$

7) Actuarial gain from interest change: 419.00 (2 - 6)

8) Actuarial loss due to salary change 450.43
(conditional on the previous change in interest).

Each scenario has two tables:

i) Demographic information used in each assessment.

ii) Information on the result of the respective actuarial liability in terms of the PBO (t+1).In The starting PBO in (t) is the same for all scenarios.

It is very common to express the tree that breaks down the obligations of actuarial Gains and Losses in percentages (%) that explain the final breakdown balance.

BALANCE	
1) Total Actuarial Loss	28.33
2) Actuarial loss due to plan experience	81.77
3) Actuarial gain due to change of assumptions	53.44
4) Actuarial gain Interest rate change	419.00
6) Actuarial loss Change Salary rate	450.43

Normalization equations (α, 1 – α)

In order to express in % the different contributions of the positions in each of its parts, we proceed to solve the following equations:

FIRST DECOMPOSITION

$$PAT = PAE + GACS$$

PAT: Total actuarial loss.

PAE: Actuarial loss plan experience.

GACS: Actuarial gain assumed changes.

At normal level with (α, 1 – α) in MBs.

28.33 = 81.77α + (1 – α) (-53.44)

α = 60.45% Plan Experience Impact.

β = (1 – α) = 39.55%

SECOND DECOMPOSITION

$$GACS = ECS + ECI$$

GACS: Current profit change assumptions.

ECI: Actuarial gain from interest rate change.

ECS: Actuarial loss from the change in the wage rate conditional on the first change in the interest.

At normal level with (α, 1 – α)

-53.44 = - 419α + (1 – α) 450.43

α = 57.96% **Actuarial gain from the change in the interest rate.**

β = (1 – α) = 42.04% **Actuarial loss due to salary change.**

7. Conclusions

The central motivation of this paper, as mentioned at the beginning, is to give a reasonable approximation in terms of the order of magnitude of each component of actuarial gains and losses in

each scenario generated. Realistically, the individual calculation can be very tedious, as verified by several expert authors in the field. The fundamental contribution of the paper is to give a very pragmatic view to determine the effects, based on the different differential components of PBO between $(t, t+1)$ and the respective expenditure of the year.

Although there are several published articles that contain very detailed calculations at the individual level (primarily in English), there is a paucity of studies at the consolidated level, and research such as ours is warranted. In the case at hand, we conclude that the total actuarial loss obtained in the final valuation is $(t, t+1)$ in the amount of 28.33 MBs, 60.45% corresponds to the experience of the plan and the remaining 39.55% corresponds to the change of assumptions. In turn, the impact/effect of the change in the interest rate translates into a profit in the order of almost 58% and the rest corresponds to the change in the wage rate.

In this assessment there was no change in staff turnover or in the mortality table used in the assessment. The latter does not influence our opinion at all in the results obtained.

Obviously individual calculations that complicate in a significant way the obtaining of the decomposition of the Gains or Losses, will give more precise and accurate results – at a cost. However, we consider that the level of approximation provided here, on a cost-benefit basis, is reasonable enough to give a quick and relatively reliable view of the final results.

SCENARIOS EVALUATED TO MEASURE PBO

APPENDICES

SCENARIO # 1:

DATE 2021

New assumptions (s = 96%, i = 108%, i_r 6%)

		ACTIVE			2330,00%	
		TOTAL			S=2330% i=4% real R=35%	
	CONCEPT	FEM	MAS	TOTAL	30/12/2020	DIF 2021/ 2020
Statistics	Population	63	113	176	166	6,02%
	Actuarial Age (Average)	40,51	41,29	41,01	42,92	-4,43%
	Current Service (Average)	4,35	6,01	5,42	8,22	-34,10%
	Comprehensive Salary (Average) (Bs.)	1.092,59	700,48	840,84	89,76	836,78%
	Payroll / Month (Bs.)	68.833,09	79.154,56	147.987,65	14.899,95	893,21%
Dif PS	PBO Diferencial (Bs.)	211.755,69	238.677,14	450.432,84	75.262,66	498,48%
	Costo por Servicio (Bs.)	36.987,68	37.361,29	74.348,97	40.977,07	81,44%
	Costo por Interés (Bs.)	218.114,99	230.217,36	448.332,36	340.826,25	31,54%
	ABO Diferencial (Bs.)	44.497,06	51.855,35	96.352,41	6.497,17	1382,99%

SCENARIO # 2:

DATE 2021

Old assumptions (s = 2330%, i = 2427%, i_r 4%), Pg = 494,80%

		ACTIVE TOTAL			2330,00% S=2330% i=4% real R=35%	
	CONCEPT	FEM	MAS	TOTAL	30/12/2020	DIF 2021/ 2020
Statistics	Population	63	113	176	166	6,02%
	Actuarial Age (Average)	40,51	41,29	41,01	42,92	-4,43%
	Current Service (Average)	4,35	6,01	5,42	8,22	-34,10%
	Comprehensive Salary (Average) (Bs.)	1.092,59	700,48	840,84	89,76	836,78%
	Payroll / Month (Bs.)	68.833,09	79.154,56	147.987,65	14.899,95	893,21%
Dif PS	PBO Diferencial (Bs.)	230.960,93	272.917,42	503.878,35	75.262,66	569,49%
	Costo por Servicio (Bs.)	116.561,37	122.875,23	239.436,60	40.977,07	484,32%
	Costo por Interés (Bs.)	1.096.552,07	1.226.516,82	2.323.068,89	340.826,25	581,60%
	ABO Diferencial (Bs.)	965,65	27.894,43	28.860,08	6.497,17	344,19%

SCENARIO # 3:

DATE 2021

Mix assumptions (s = 96%, i = 2427,20%, i_r 1189,39%)

		ACTIVE TOTAL			2330,00% S=2330% i=4% real R=35%	
	CONCEPT	FEM	MAS	TOTAL	30/12/2020	DIF 2021/ 2020
Statistics	Population	63	113	176	166	6,02%
	Actuarial Age (Average)	40,51	41,29	41,01	42,92	-4,43%
	Current Service (Average)	4,35	6,01	5,42	8,22	-34,10%
	Comprehensive Salary (Average) (Bs.)	1.092,59	700,48	840,84	89,76	836,78%
	Payroll / Month (Bs.)	68.833,09	79.154,56	147.987,65	14.899,95	893,21%
Dif PS	PBO Diferencial (Bs.)	2.320,60	29.115,45	31.436,05	75.262,66	-58,23%
	Costo por Servicio (Bs.)	7.262,54	33.878,23	41.140,77	40.977,07	0,40%
	Costo por Interés (Bs.)	- 170.558,83	98.964,07	- 71.594,76	340.826,25	-121,01%
	ABO Diferencial (Bs.)	965,65	27.894,43	28.860,08	6.497,17	344,19%

SCENARIO # 4:

DATE 2020

New assumptions (s = 96%, i = 108%, i_r 6%)

		ACTIVE TOTAL			791,61% S=791,61% i=4% real R=35%	
	CONCEPT	FEM	MAS	TOTAL	30/12/2019	DIF 2020/ 2019
Statistics	Population	61	105	166	181	-8,29%
	Actuarial Age (Average)	42,77	43,00	42,92	42,28	1,50%
	Current Service (Average)	8,53	8,04	8,22	8,23	-0,13%
	Comprehensive Salary (Average) (Bs.)	94,10	87,23	89,76	1,10	8043,59%
	Payroll / Month (Bs.)	5.740,30	9.159,64	14.899,95	199,50	7368,71%
Dif PS	PBO Diferencial (Bs.)	24.954,35	41.595,35	66.549,71	1.023,95	6399,34%
	Costo por Servicio (Bs.)	3.695,75	5.624,06	9.319,80	181,81	5026,04%
	Costo por Interés (Bs.)	12.958,91	19.835,20	32.794,11	1.279,48	2463,09%
	ABO Diferencial (Bs.)	3.352,46	10.115,86	13.468,32	133,79	9967,10%

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