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Original Paper

Approximate Method for Calculation of Actuarial Liabilities under IAS 19 with the Unit Credit Method of Projected Benefit

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Background

The Projected Benefit method to determine the Actuarial Liability of different defined Benefit plans requires projecting future benefits prorated for the years accumulated as of the valuation date. In the case of a regular Retirement plan, where there is a normal retirement reference age and possibly 2 or 3 early retirement ages, the attribution of the Benefit does not represent a major calculation problem in the sense that the Benefit adjustments are simple prorations, for those 2 or 3 future ages.

However, when dealing with defined benefits type LUMP-SUM "Severance plans", where the plan considers multiple exits from the valuation date until a final exit age, the calculation becomes a little complicated to determine the present expected value VEP of future payments, given that the traditional recursiveness of the formulas is lost in the sense that the $VEP_t \neq (VEP_{t-1})(a) + (b)$, being constants.

An important case study is when you have a Benefit (B_t) of type $B_t = S_t t$, that is, the salary in t times the creditable service in t and it turns out that for some reason the t maximum (t) is limited to a constant value k in the future time: That is, from k, the computable Benefit will be; $B_t = (k)(S_t)$; that is, if $t \geq k$.

This paper addresses a mechanism to approximate it recursively, making some adjustments tha0t lighten and optimize the calculation in a fairly efficient way, with a **small margin of error.**

Keywords: Defined benefits, IAS19, FASB87, Projected Benefit Method, Actuarial Models, Liabilities and Actuarial Costs. Severance Benefit, Lump Sums

1. Projected Benefit Method (PUC)

The calculation of the liability is generally given by the calculation of the PBO Projected Benefit Obligation. The PBO is a function of B_t and it must be prorated as follows:

B_t: Benefit in t

S_t: Salary in t

The Benefit according to the requirements of a very common Benefit plan can be given by the following structure:

$$B_t = \begin{cases} (S_t)(t) & Si\ t \leq 25 \\ (S_t)(25) & Si\ t > 25 \end{cases} \qquad \textbf{(1)}$$

With $k = 25$

Prorate Sequence $\left\{{}^{t}/_{t}, {}^{t}/_{(t+1)}, {}^{t}/_{t+2}, \cdots\right\} \to \{FD_t\}$

That is, the Benefit is limited to 25 years of creditable service, the above tells us that after 25 years the Benefit grows only by salary, but not by time. The (PBO) as usual is calculated as:

$PBO_t = (S_t)(t)(V^t)(FD_t)(P_t)$, where $V = \frac{(1+S)}{(1+i)}$ and P_t, the conditional probability of surviving until t and exiting in $t+1$ **(2)**

FD_t: Accrual factor or benefit attribution (Proration).

The above works very well, using recurring calculations $PBO_{t-1} = (a)PBO_{t-1} + (b)$ without any problem of any nature. If we imagine an increasing payment of the $B_t$salary and time benefit, the calculation is almost identical to that of life insurance:

$$A_x = q_x\left(\frac{1+s}{1+i}\right) + \left(\frac{1+s}{1+i}\right)^2 P_x q_{x+1} + \left(\frac{1+s}{1+i}\right)^3 {}_2P_x q_{x+2} + \cdots \qquad \textbf{(3)}$$

$$A_{x+1} = q_{x+1}\left(\frac{1+s}{1+i}\right) + \left(\frac{1+s}{1+i}\right)^2 {}_1P_{x+1} q_{x+2} + \left(\frac{1+s}{1+i}\right)^3 {}_2P_x q_{x+2} + \cdots \qquad \textbf{(4)}$$

The recurrence is fulfilled, if we relate the previous formulas, it is observed that both series comply with:

$$A_x = vq_x + vP_x A_{x+1} \qquad \textbf{(5)}$$

The previous identity allows the calculations of the present expected values to be made very quickly and efficiently.

On the contrary, when we have the restriction of 25 years, the above is not exactly met, but it can be reasonably approximated in aggregate terms, especially in large amounts of data.

This approach goes through the following restrictions:

1. **Is the demographics of the company** are significantly concentrated in relatively low ages and services.

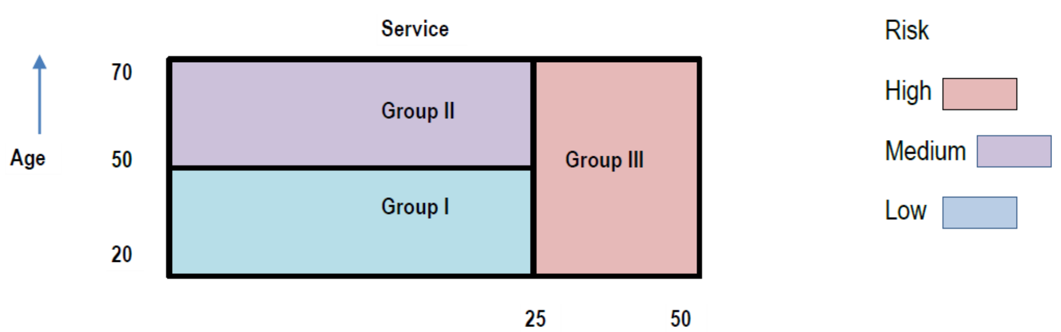

Figure 1

a. **From the previous graph and table 1,** it is important to know that there are 2,527 employees under 50 years of age.

b. There are 26 employees who have been in service for more than 25 years. with age between (50,70).

c. Service over 30 years only 2 employees.

The detailed previous distribution is found in the annexes. Generally, this is a bivariate distribution with a large concentration of personnel in Group I and very few in Group II and III.

Table 1

Group statistics				Totales
Group	Ages	Service	#	%
Group I	19-50	≤ 25	2299	89%
Group II	>50	≤ 25	268	10%
Group III	50-70	>25	26	1%
			2593	**100%**

2. **If the entire group I is calculated,** with the approximation using the recurrence explained above, the total error of the aggregate calculation does not exceed 1%.

3. On the other hand, when using rates of personnel exits with payment for dismissals, resignations and death under a relatively dense exponential model in exits at early ages, the PBO (Projected Benefit Obligation) or the VEP (Present Expected Value) with or without constrains are practically the same.

2. Company Taken as a Sample. Company Characteristics

The company is made up of 2,593 employees and is distributed as follows:

a. 12% female employees.

b. 88% male employees, with an average age of 36 years, a service of 5.06 years with an monthly average reference salary of 26,337.11, for the entire population (male - female).

All the data from which the samples were taken to make the comparative calculations of the PBO Actuarial Liability at the border points indicated above, figure 1 obeyed the following hypotheses:

1. Actuarial assumptions:

a. Salary increase rate (s %): 3.49

b. Nominal interest rate (i%): 6.75%

c. Equivalent real interest rate on salary (Γ%): 3.15%

d. Staff exits/turnover rates:

i. d:89.57%

ii. r:7.93%

iii. m:0.29%

iv. other causes:2.21%

e. Age of leaving the company x: $70\ years$

f. d: dismissals
   r: resignations
   m: death

2. Mathematical Model (PUC)

The model is composed of the following variables:

1. x: Age of the employee.
2. T: Years of service.
3. TB: Years of service with the restriction of $K = 25\ años$

4. FD: Accrual factor or profit attribution.

   to. According to the projected profit method.

5. V: Real discount;; $V = \left(\frac{1+\delta}{1+i}\right)$
6. t: sequential time to determine the powers of V.
7. $l_x(T)$: Age survivors x for all contingencies T.
8. h. $d_x(T)$: Estimated expected exits by age.

   a. For that contingency

9. $PROB$: Conditional probability of exits.
10. PBO_1: Obligation for projected benefits using T.
11. PBO_2: Obligation for projected benefits using TB.

The actuarial mathematical model to calculate that PBO_{total} of an individual j, characterized by an age x and a service, T is given by:

i. Without restrictions

$PBO_x^1 = \sum T_x FD_x V^t\ PROB_x$ for all x in $(x, 70)$ and $t(1, 70 - x)$ **(6)**

ii. With restriction

$PBO_x^2 = \sum TB_x FD_x V^t\ PROB_x$ for all $x\ and\ t$ **(7)**

$PBO\ ACUM$: Cumulative values by age for each $PBO1$ and $PBO2$

$Diferencial$: Differential between $(PBO1) - (PBO2)$ to evaluate the differences by age if they are significant.

Case #1: Employee of current age $x = 20$ and $T = 20\ años$

Results in terms of $PBO_1 = PBO_2 = \Delta PBO \cong 0$ **(8)**

Almost zero error level: 0.0040[1]

Calculation model example of an employee with a starting age of 43 years and 20 years of service.

[1] $(\Delta PBO)/PBO_2$

Table 2

X	T	TB	FD	t	V	lx (T)	dx (T)	PROB	PBO 1	PBO ACUM 1	PBO 2	PBO ACUM 2	DIFRENCIAL
43	20	20	1,00	1	0,96950223	0,0000	0,0000	0,4131	8,0104	8,0104	8,0104	8,0104	-
44	21	21	0,95	2	0,93993457	0,0000	0,0000	0,2364	4,4448	12,4552	4,4448	12,4552	-
45	22	22	0,91	3	0,91126866	0,0000	0,0000	0,1377	2,5096	14,9648	2,5096	14,9648	-
46	23	23	0,87	4	0,88347699	0,0000	0,0000	0,0815	1,4408	16,4056	1,4408	16,4056	-
47	24	24	0,83	5	0,85653291	0,0000	0,0000	0,0491	0,8404	17,2460	0,8404	17,2460	-
48	25	25	0,80	6	0,83041057	0,0000	0,0000	0,0300	0,4977	17,7437	0,4977	17,7437	-
49	26	25	0,77	7	0,80508489	0,0000	0,0000	0,0186	0,2991	18,0428	0,2876	18,0313	0,0115
50	27	25	0,74	8	0,7805316	0,0000	0,0000	0,0117	0,1822	18,2250	0,1687	18,2000	0,0250
51	28	25	0,71	9	0,75672712	0,0000	0,0000	0,0074	0,1125	18,3376	0,1005	18,3005	0,0371
52	29	25	0,69	10	0,73364863	0,0000	0,0000	0,0048	0,0704	18,4080	0,0607	18,3612	0,0468
53	30	25	0,67	11	0,71127398	0,0000	0,0000	0,0031	0,0446	18,4525	0,0371	18,3983	0,0542
54	31	25	0,65	12	0,68958171	0,0000	0,0000	0,0021	0,0286	18,4811	0,0230	18,4213	0,0597
55	32	25	0,63	13	0,668551	0,0000	0,0000	0,0014	0,0185	18,4996	0,0145	18,4358	0,0638
56	33	25	0,61	14	0,64816169	0,0000	0,0000	0,0009	0,0121	18,5117	0,0092	18,4450	0,0667
57	34	25	0,59	15	0,6283942	0,0000	0,0000	0,0006	0,0080	18,5197	0,0059	18,4509	0,0688
58	35	25	0,57	16	0,60922958	0,0000	0,0000	0,0004	0,0054	18,5251	0,0038	18,4547	0,0704
59	36	25	0,56	17	0,59064943	0,0000	0,0000	0,0003	0,0036	18,5287	0,0025	18,4572	0,0715
60	37	25	0,54	18	0,57263594	0,0000	0,0000	0,0002	0,0025	18,5312	0,0017	18,4589	0,0723
61	38	25	0,53	19	0,55517182	0,0000	0,0000	0,0002	0,0017	18,5329	0,0011	18,4600	0,0729
62	39	25	0,51	20	0,53824031	0,0000	0,0000	0,0001	0,0012	18,5340	0,0008	18,4608	0,0733
63	40	25	0,50	21	0,52182518	0,0000	0,0000	0,0001	0,0008	18,5349	0,0005	18,4613	0,0736
64	41	25	0,49	22	0,50591068	0,0000	0,0000	0,0001	0,0006	18,5355	0,0004	18,4616	0,0738
65	42	25	0,48	23	0,49048153	0,0000	0,0000	0,0000	0,0004	18,5359	0,0002	18,4619	0,0740
66	43	25	0,47	24	0,47552293	0,0000	0,0000	0,0000	0,0003	18,5362	0,0002	18,4621	0,0741
67	44	25	0,45	25	0,46102054	0,0000	0,0000	0,0000	0,0002	18,5364	0,0001	18,4622	0,0742
68	45	25	0,44	26	0,44696044	0,0000	0,0000	0,0000	0,0002	18,5365	0,0001	18,4623	0,0743
69	46	25	0,43	27	0,43332915	0,0000	0,0000	0,0000	0,0001	18,5367	0,0001	18,4623	0,0743
70	47	25	0,43	28	0,42011357	0,0000	0,0000	0,0000	0,0004	18,5370	0,0002	18,4625	0,0745

When the PBO_2 cumulative figure is graphed, its quasi-convergence is observed after 10 years of service.

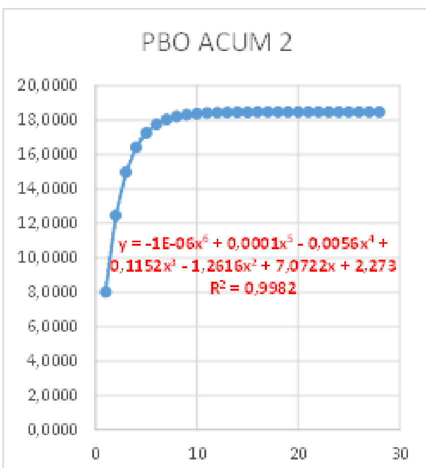

Chart 2

3. Simulated Scenarios

In order to explore the error levels, a group of individual calculations were generated for the boundaries of each of the groups described above.

A large group of cases were chosen from the real valuation of a company that in terms of age and service were close to the border previously indicated in the graph.

Table 3

Edad / Servicio	20	21	22	23	24	25	26	28	29	31	33	38
43	0,40%					5,07%						
44				1,86%	3,10%							
47					3,62%							
48	0,68%					6,12%	10,14%		22,20%	30,23%		
50		1,20%										
52					4,57%			19,08%		31,14%		
54	1,12%											
55			2,44%	3,53%		7,69%						
56		1,81%										
58			2,81%									
59		2,06%			5,90%							
60	1,45%		2,98%			8,70%						
61							12,85%				41,01%	
62	1,29%		2,96%	4,29%					24,98%			
64	0,65%					8,67%		20,74%				
66										31,75%		
68												57,21%

From the previous table it follows:

1. All cases with an age less than or equal to 43 years and 20 years of service, the error is null.

2. In the age group between 43 and 70 years old with less than 23 years old, the error does not reach 5%.

3. The number of employees with service greater than 30 years is 2 and between 25 and 30 years for all ages is 24. These two groups indicated above would definitely be those where a greater estimation error could be made, specifically those over 60 years of age with more than 25 years of service, and in any case, it would be an overestimation of the liability that is easily fixed by applying a factor to those cases in the order of 80% to 90%.

4. Conclusions and Recommendations

1. There is no doubt that the work of the actuarial valuation is reduced in a very important way by calculating GROUP I in a recurring quasi-exact manner with zero errors.

2. Cases outside GROUP I can be approximated in the same way on a recurring basis with an adjustment factor that probably slightly reduces the liability that was calculated without the restriction. If for some reason this overestimation was not acceptable, then the PBO of each of these employees would be calculated exactly; but obviously, the number of people or employees would be very small compared to the total mass of workers.

3. We must not forget, on the other hand, that the actuarial valuation is nothing more than an estimation and probably within a consolidation environment of PBO_{TOTAL}, an interval of $PBO_T \pm 10\%$, in our opinion would be more than reasonable.

4. Obviously, the above cannot be applied to all cases, particularly in those where the density of personnel is strongly biased towards Group III. Generally, in companies, it is not common, but it could happen. In those cases, it would of course not be advisable to make such an approach. When we talk about a number of employees as large as the company that was taken as a sample, it is almost illogical

that there is a very large number of people in GROUP III, if not impossible.

5. However, even in the case of GROUP III, if the creditable services are not high, possibly less than 15 years, then the approximation could also be reasonable.

6. The level of errors in the cases evaluated are frankly insignificant, in our opinion, immaterial. Therefore, the fact that an actuarial liability is taken with $\pm 10\%$, should not be worrying to anyone.

7. Obviously, the results of this study are subject to the observed demographics of the company, and its actuarial assumptions or hypotheses. Each company has its own demographic profile, however, in terms of characterization of companies by their bivariate distribution of the number of employees by age and years of service, this profile is probably the one most generally observed in the vast majority of companies. at least in Latin America. If the net valuation interest rate rises, the error tends to decrease, given that in general the higher the real spread rate, the smaller the future present values.

In conclusion, the valuation of these types of benefits, under the treatment of a life insurance premium on a recurring basis, is completely applicable to the previous valuation of the aforementioned contingent benefits.

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Anexo I

Normal rotations					
	Mortality				
x	Male	Female	Dissmissals	Resigns	Other causes
18	0,047%	0,019%	26,301%	4,836%	47,181%
19	0,049%	0,019%	25,632%	4,713%	45,981%
20	0,038%	0,019%	24,981%	4,594%	44,812%
21	0,039%	0,020%	24,346%	4,477%	43,673%
22	0,041%	0,021%	23,727%	4,363%	42,563%
23	0,042%	0,023%	23,124%	4,252%	41,481%
24	0,044%	0,024%	22,536%	4,144%	40,426%
25	0,046%	0,025%	21,963%	4,039%	39,399%
26	0,049%	0,027%	21,404%	3,936%	38,397%
27	0,051%	0,028%	20,860%	3,836%	37,421%
28	0,054%	0,030%	20,330%	3,738%	36,469%
29	0,057%	0,032%	19,813%	3,643%	35,542%
30	0,061%	0,034%	19,309%	3,551%	34,639%
31	0,065%	0,036%	18,819%	3,460%	33,758%
32	0,069%	0,039%	18,340%	3,372%	32,900%
33	0,073%	0,041%	17,874%	3,287%	32,063%
34	0,079%	0,044%	17,419%	3,203%	31,248%
35	0,086%	0,048%	16,977%	3,122%	30,454%
36	0,091%	0,050%	16,545%	3,042%	29,680%
37	0,097%	0,054%	16,124%	2,965%	28,925%
38	0,104%	0,057%	15,714%	2,890%	28,190%
39	0,113%	0,062%	15,315%	2,816%	27,473%
40	0,124%	0,067%	14,926%	2,745%	26,775%
41	0,137%	0,072%	14,546%	2,675%	26,094%
42	0,153%	0,078%	14,176%	2,607%	25,431%
43	0,172%	0,084%	13,816%	2,541%	24,784%
44	0,193%	0,092%	13,465%	2,476%	24,154%
45	0,218%	0,101%	13,122%	2,413%	23,540%
46	0,247%	0,112%	12,789%	2,352%	22,941%
47	0,279%	0,124%	12,464%	2,292%	22,358%
48	0,314%	0,137%	12,147%	2,234%	21,790%
49	0,351%	0,151%	11,838%	2,177%	21,236%
50	0,391%	0,165%	11,537%	2,121%	20,696%
51	0,432%	0,179%	11,244%	2,068%	20,170%
52	0,476%	0,195%	10,958%	2,015%	19,657%
53	0,520%	0,212%	10,679%	1,964%	19,157%
54	0,566%	0,232%	10,408%	1,914%	18,670%
55	0,613%	0,254%	10,143%	1,865%	18,196%
56	0,662%	0,280%	9,885%	1,818%	17,733%
57	0,714%	0,310%	9,634%	1,772%	17,282%
58	0,772%	0,344%	9,389%	1,726%	16,843%
59	0,838%	0,382%	9,150%	1,683%	16,415%
60	0,916%	0,424%	8,918%	1,640%	15,997%
61	1,006%	0,470%	8,691%	1,598%	15,591%
62	1,113%	0,521%	8,470%	1,558%	15,194%
63	1,239%	0,577%	8,255%	1,518%	14,808%
64	1,387%	0,639%	8,045%	1,479%	14,432%
65	1,559%	0,706%	7,840%	1,442%	14,065%
66	1,758%	0,782%	7,641%	1,405%	13,707%
67	1,980%	0,868%	7,447%	1,369%	13,359%
68	2,223%	0,970%	7,257%	1,335%	13,019%
69	2,482%	1,092%	7,073%	1,301%	12,688%
70	2,753%	1,239%	6,893%	1,268%	12,365%